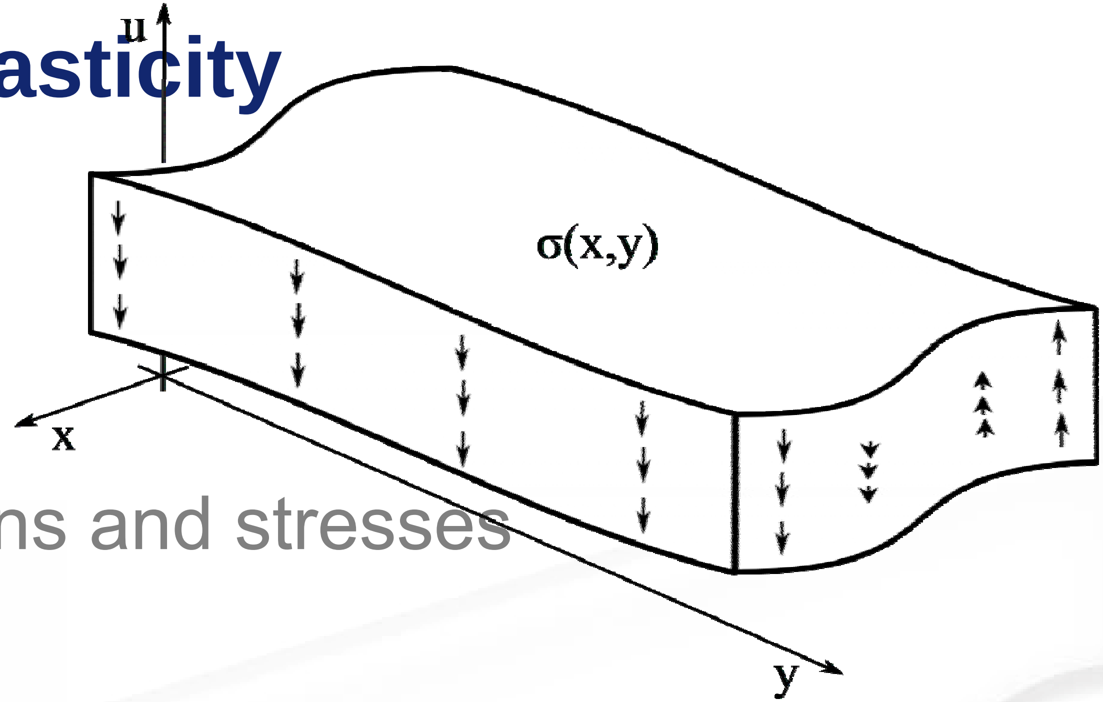
Part III – Local stress fields

- 1) crack tip stresses
  - antiplane crack
  - stress intensity factor
  - energy release rate
- 2) adhesive contact

1a) Antiplanar elasticity



- Elastic deformations and stresses

$$\bar{\epsilon} = \nabla u(x, y)$$

$$\bar{\sigma} = \mu \bar{\epsilon}$$

- Equilibrium

$$\mathrm{div}(\bar{\sigma}) = \mu \triangle (u)$$

Tip – Mode III

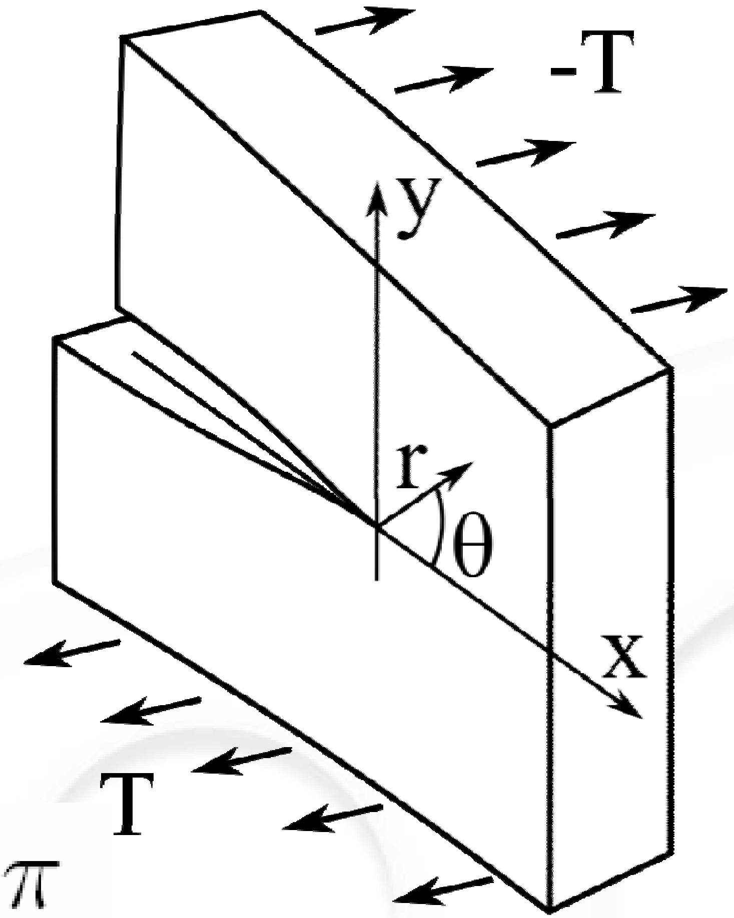
- Solutions

$$u = \Im m(\Omega)$$

with $\Omega = A z^{\alpha}$

- Boundary conditions

$$\sigma_y = 0 \text{ for } \theta = \pi$$



1b) Crack tip – Solution(s)

- General solution

$$\alpha = \frac{1}{2} + n$$

- Dominant term

  Stress intensity factor

$$u = \frac{K}{\mu} r^{\frac{1}{2}} \sin\frac{\theta}{2}$$

$$\sigma_y = \frac{K}{2} r^{-\frac{1}{2}} \cos\frac{\theta}{2}$$

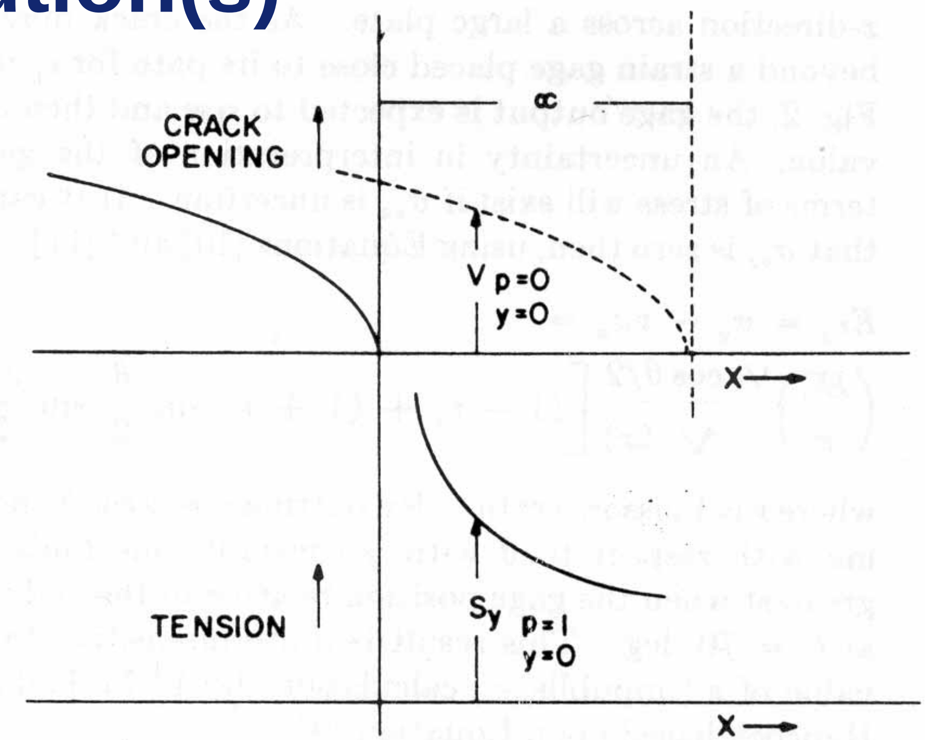
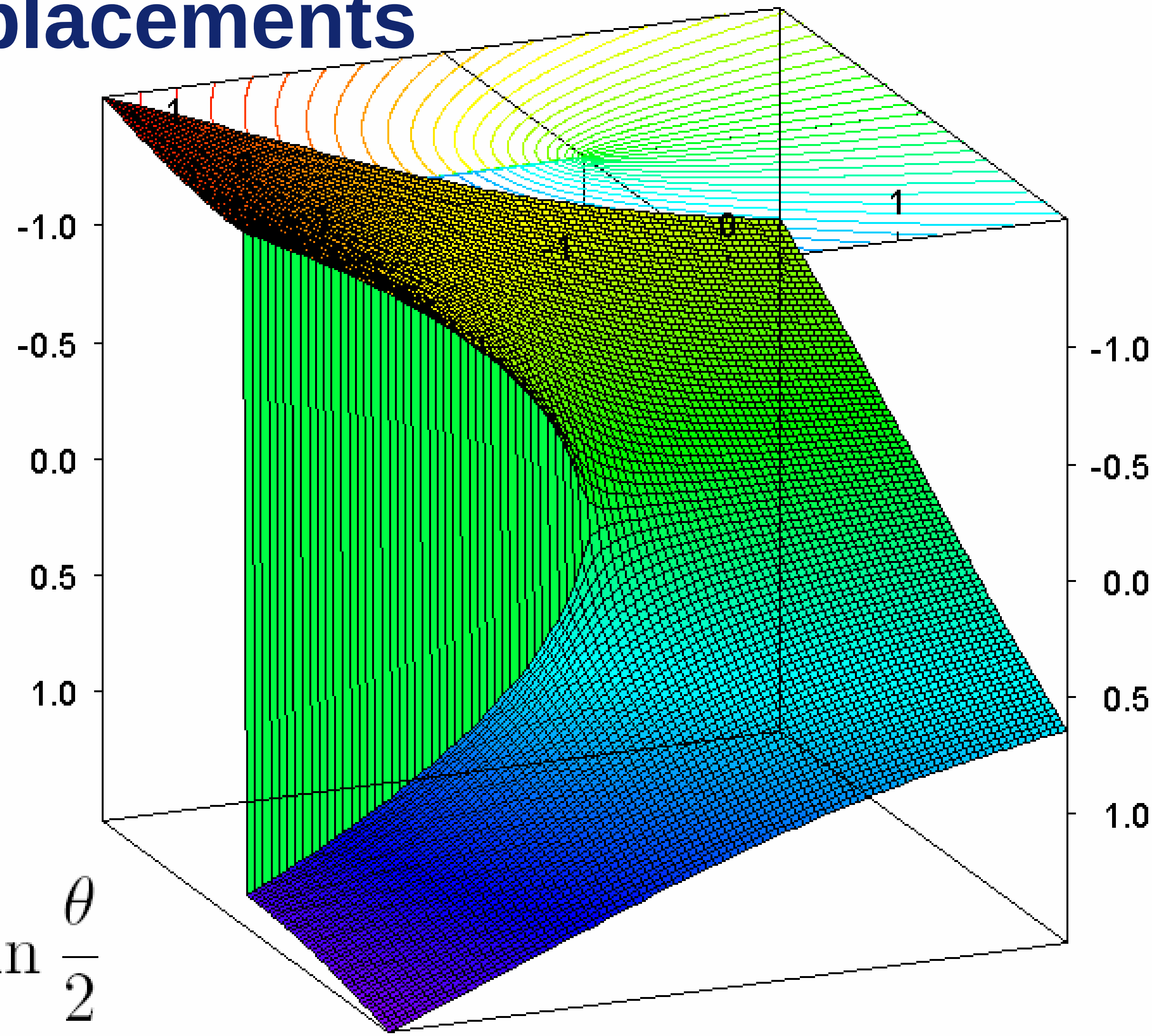


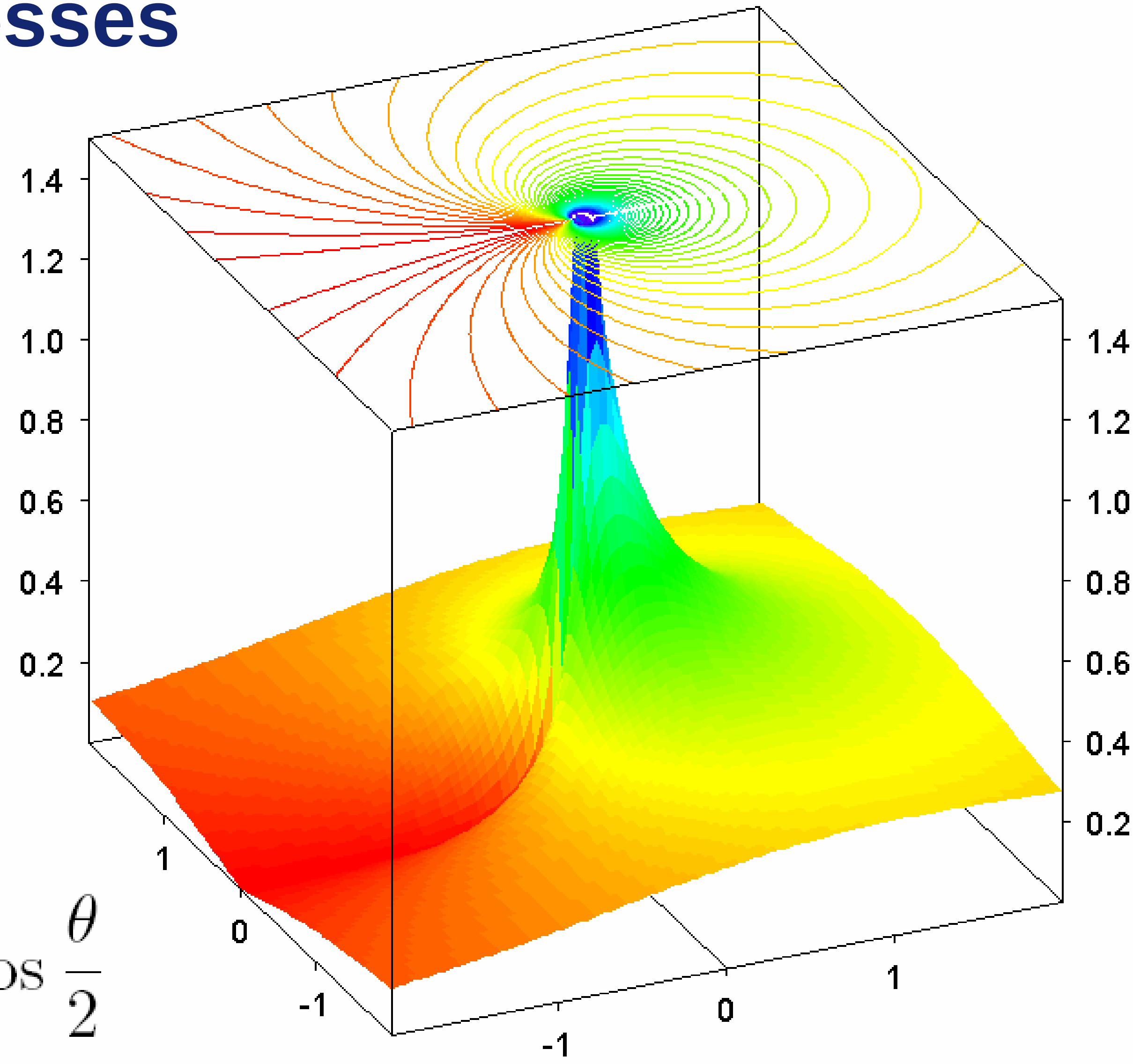
FIG. 3 LINEAR-ELASTIC-THEORY CRACK OPENINGS AND STRESSES NEAR END OF A CRACK

Crack tip displacements



$$u = \frac{K}{\mu} r^{\frac{1}{2}} \sin\frac{\theta}{2}$$

Crack tip stresses



$$\sigma_y = \frac{K}{2} r^{-\frac{1}{2}} \cos\frac{\theta}{2}$$

1c) Energy release rate and stress intensity factor

$$dU_{el} \quad = \quad \frac{b}{2}\int_0^{da} \sigma\,[u(\pi) - u(-\pi)]\,dr$$

$$\mathcal{G} = \frac{dU_{el}}{dA} = \frac{dU_{el}}{bda}$$

$$\mathcal{G} = \frac{\pi K^2}{4\mu}$$

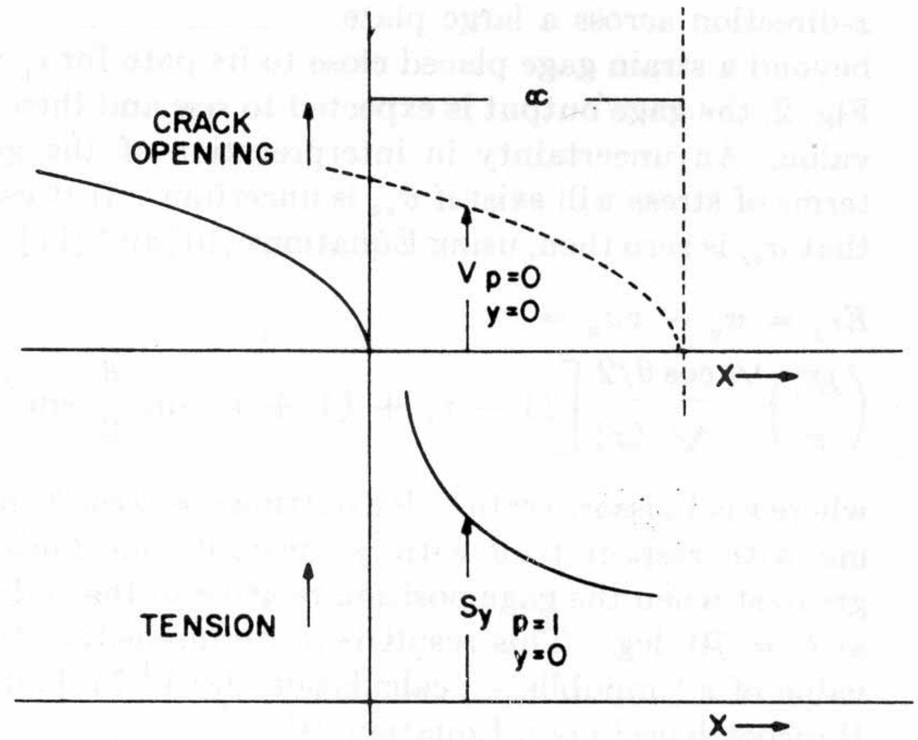


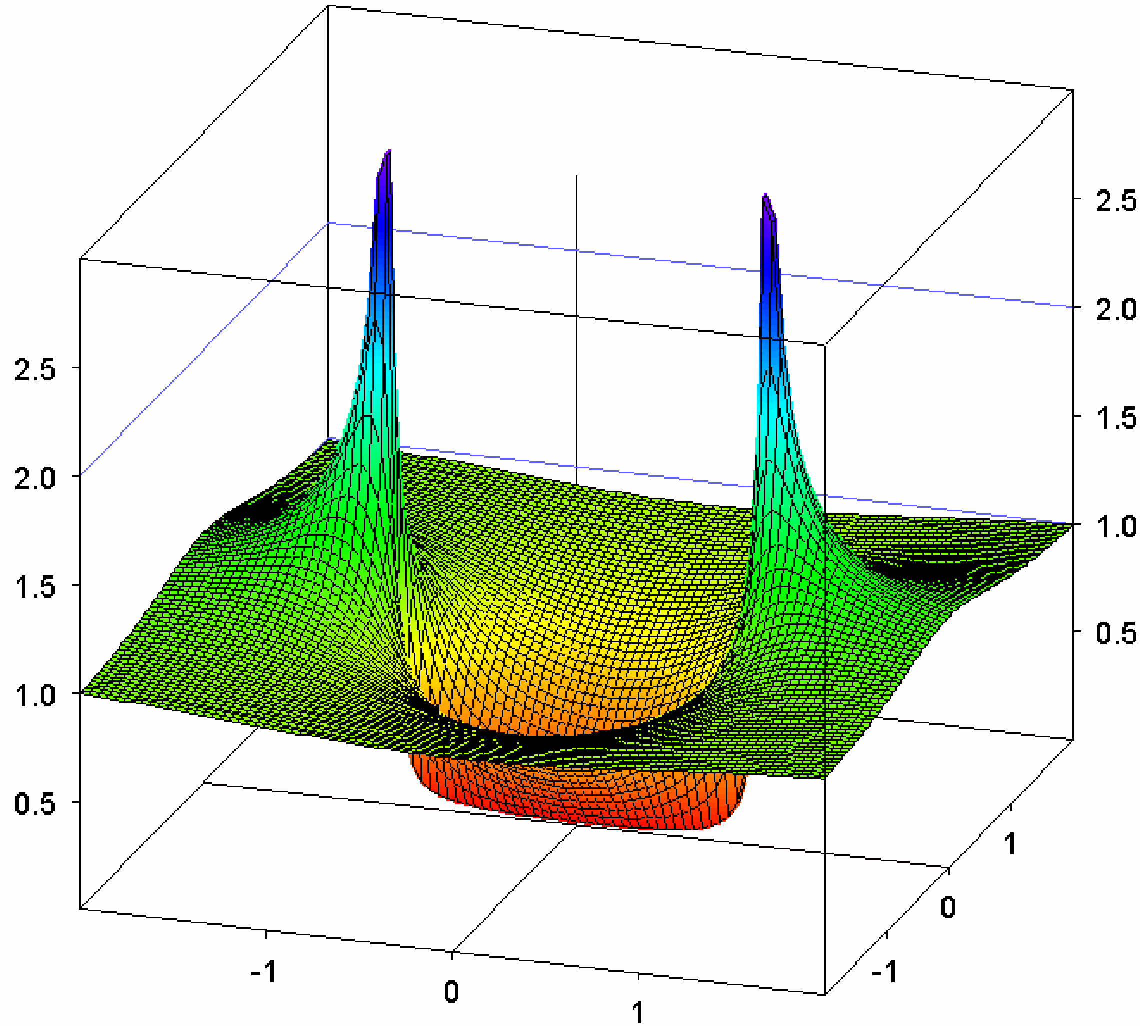
FIG. 3 LINEAR-ELASTIC-THEORY CRACK OPENINGS AND STRESSES NEAR END OF A CRACK

Stress intensity factor and remote loading

$$u = B\Re e(\sqrt{a^2 - z^2})$$

$$K = T\sqrt{2a}$$

$$\mathcal{G} = \frac{\pi}{2}\frac{aT^2}{\mu}$$



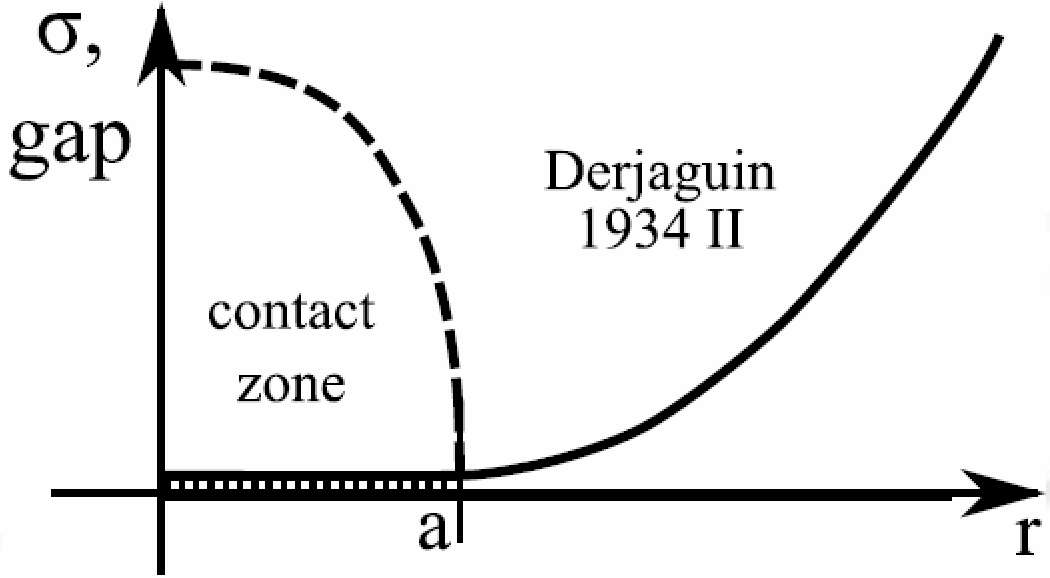
1920: A. Griffith

- In 1915 he was accepted by the Royal Aircraft Factory as a trainee, before joining the Physics and Instrument Department the following year […]
- In 1926 he wrote his classic paper, *An Aerodynamic Theory of Turbine Design*. In it, he foresaw the advantages in employing an axial gas turbine engine to drive a propeller…

http://www.cmse.ed.ac.uk/MSE3/Topics/MSE2-05/MSE2-Griffith.pdf

http://100.rolls-royce.com/people/view.jsp?id=116

2) Adhesive contact – beyond Derjaguin 1934 Part II



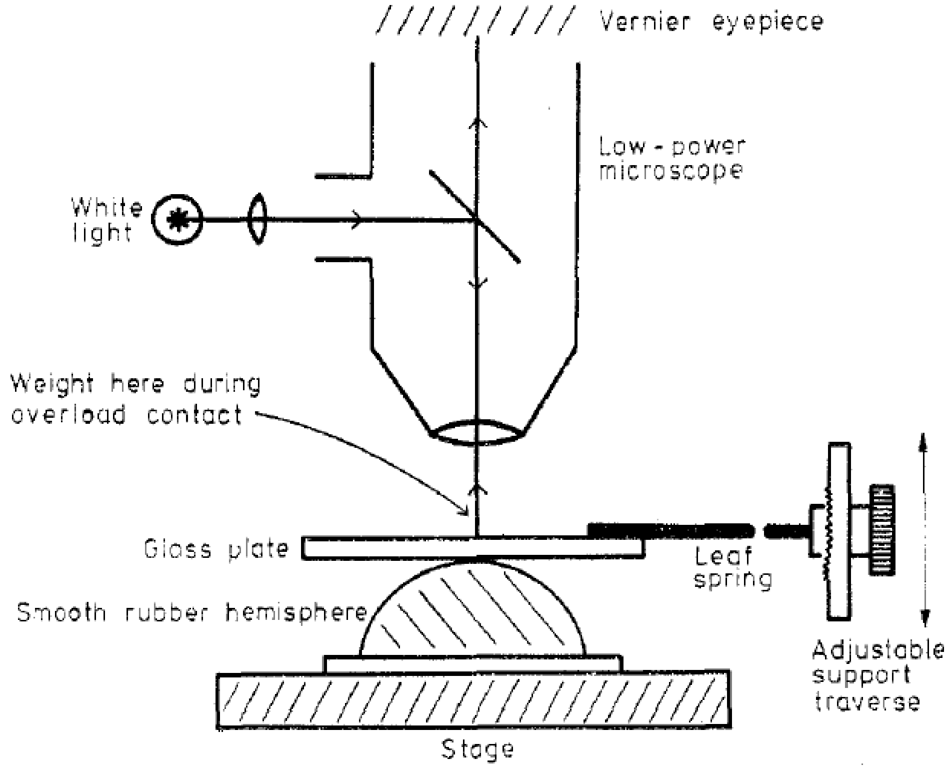
$$\mathcal{E}(\delta) = \mathcal{E}_H(\delta) - w(\pi a^2)$$

- Pull out force

$$F_{pullout,D} = -\pi R w$$

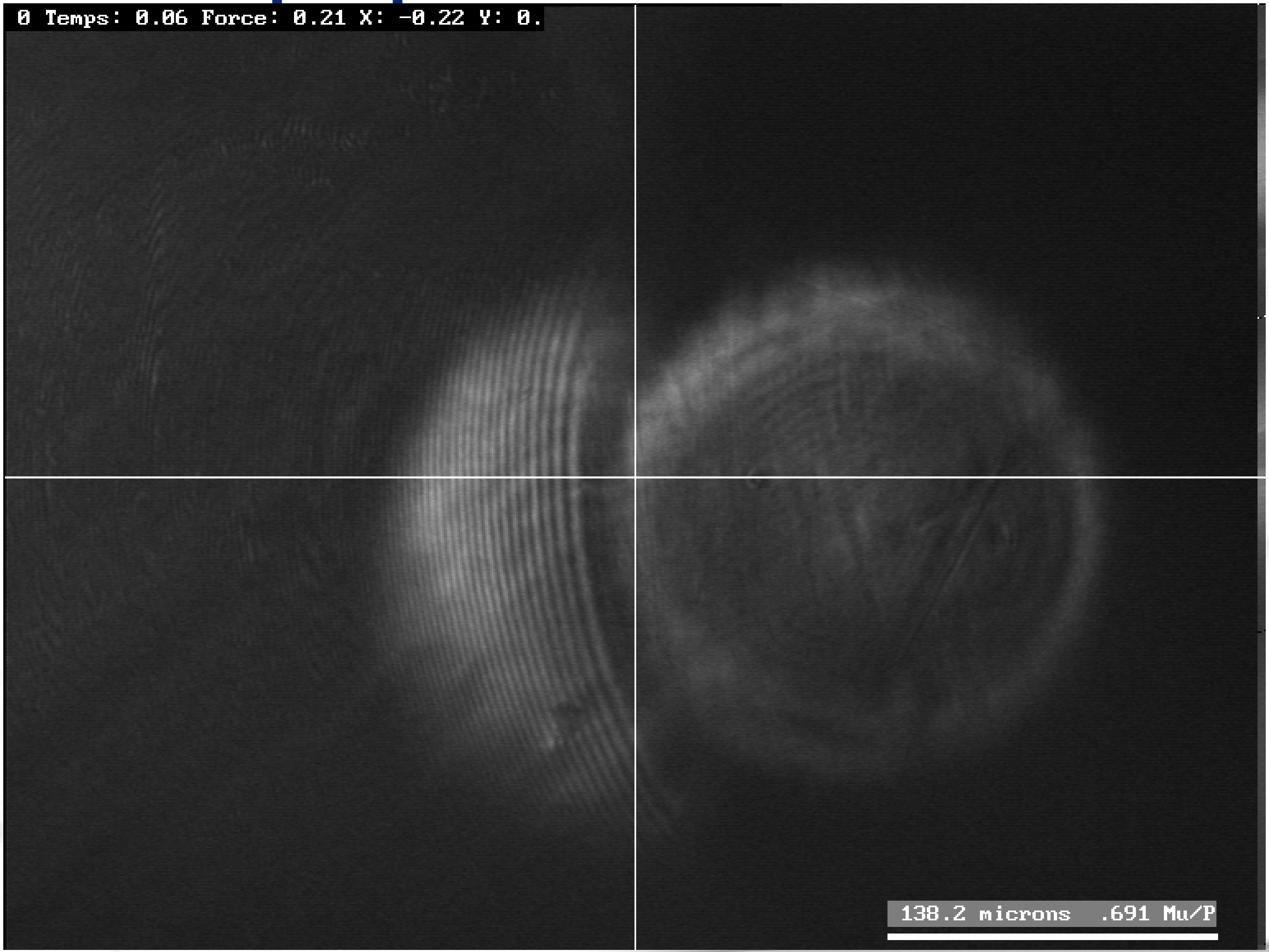
2a) 1971: JKR

- research on rubber contact at the Physics Dpt at Cambridge sponsored by the Malaysian Rubber Institute



Surface charge contribution in rubber adhesion and friction, A. Roberts Journal of Physics D: Applied Physics 10, 1977, 1801

Macroscopic picture



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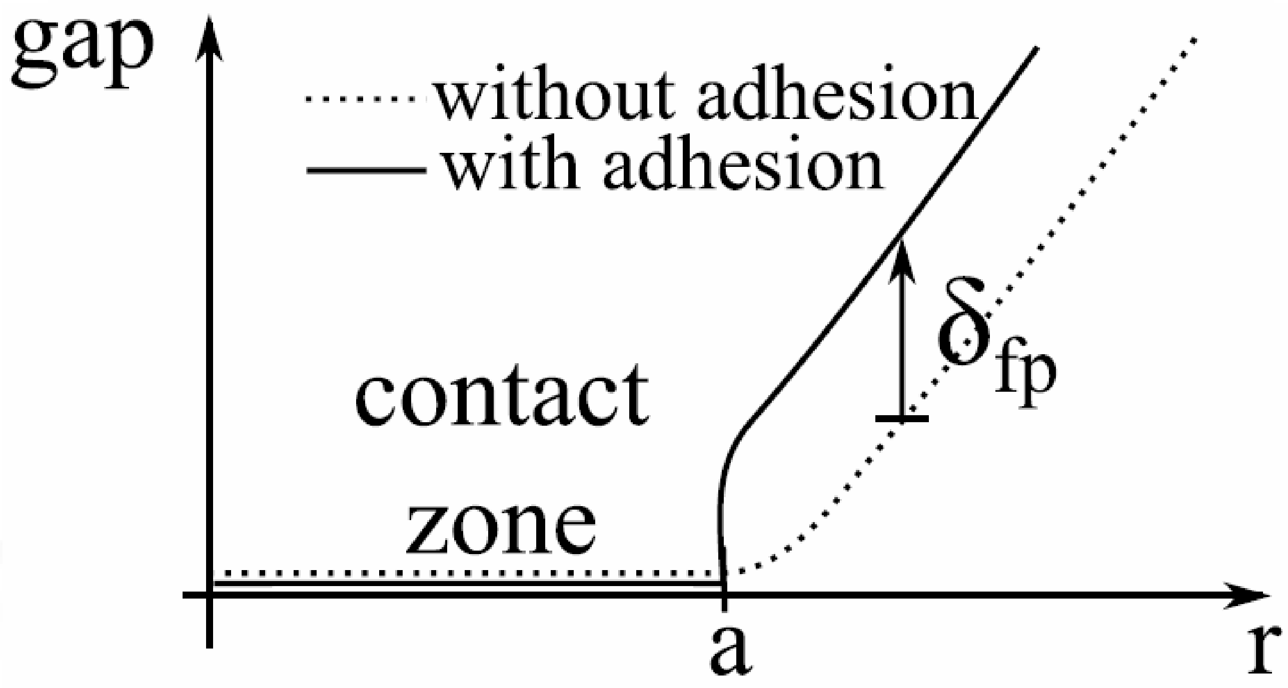
Contact of spheres – impact of adhesion



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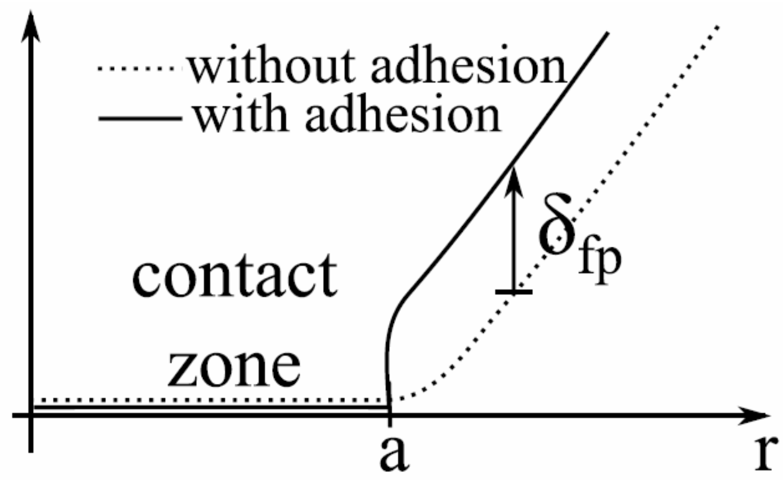
JKR (1971)



- displacement and force

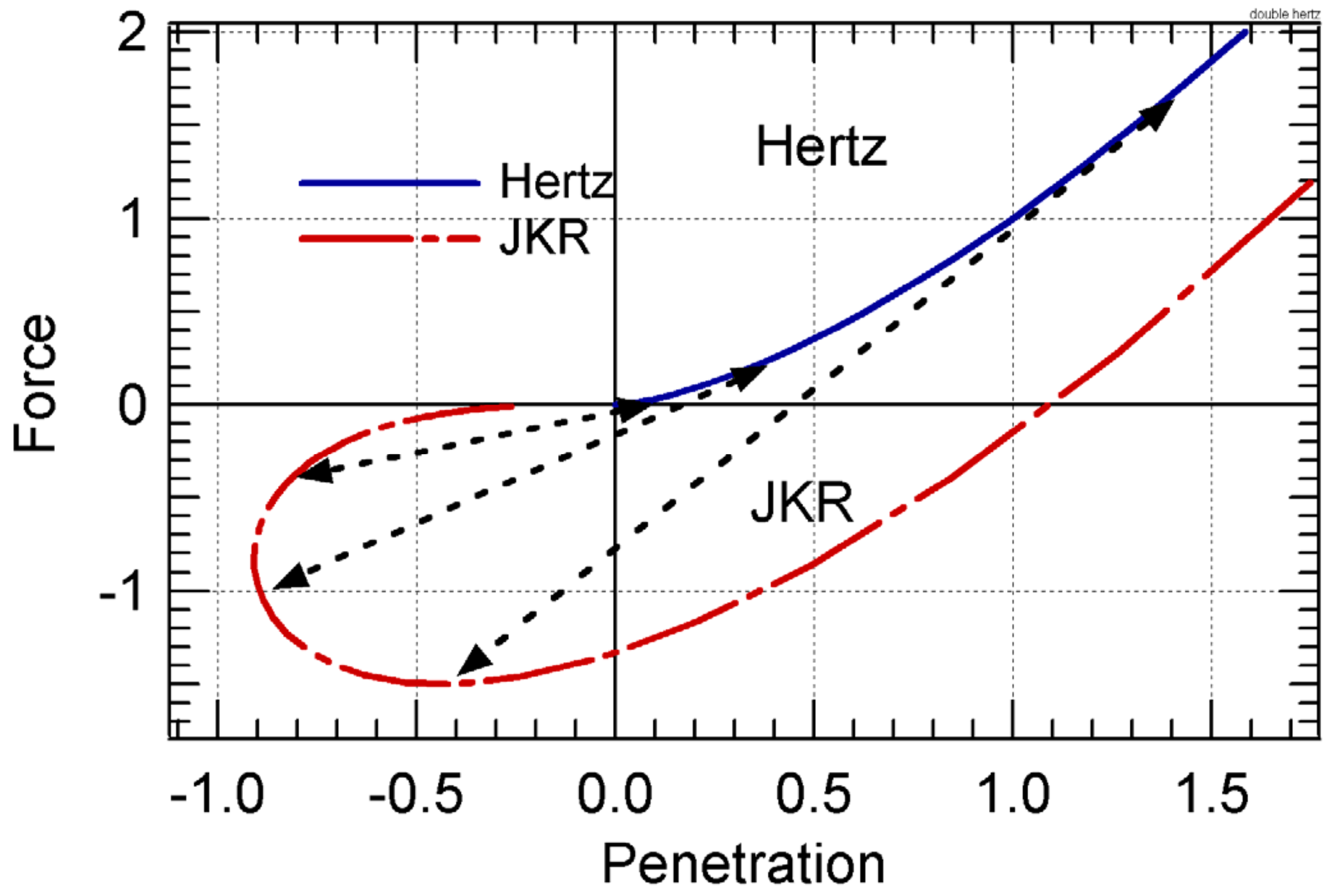
$$\delta(a) = \delta_H(a) + \delta_{fp}$$

$$F(a) = F_H(a) + F_{fp}(a)$$

with

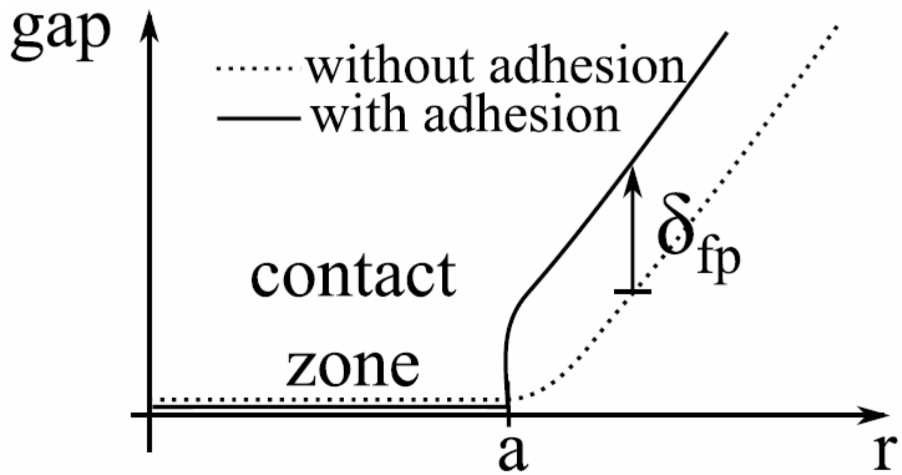
$$F_{fp}(a) = \delta_{fp} S(a)$$

Force plots



double hertz
2
1
0
-1
Force
Hertz
JKR
Hertz
JKR
-1.0
-0.5
0.0
0.5
1.0
1.5
Penetration

JKR (1971)



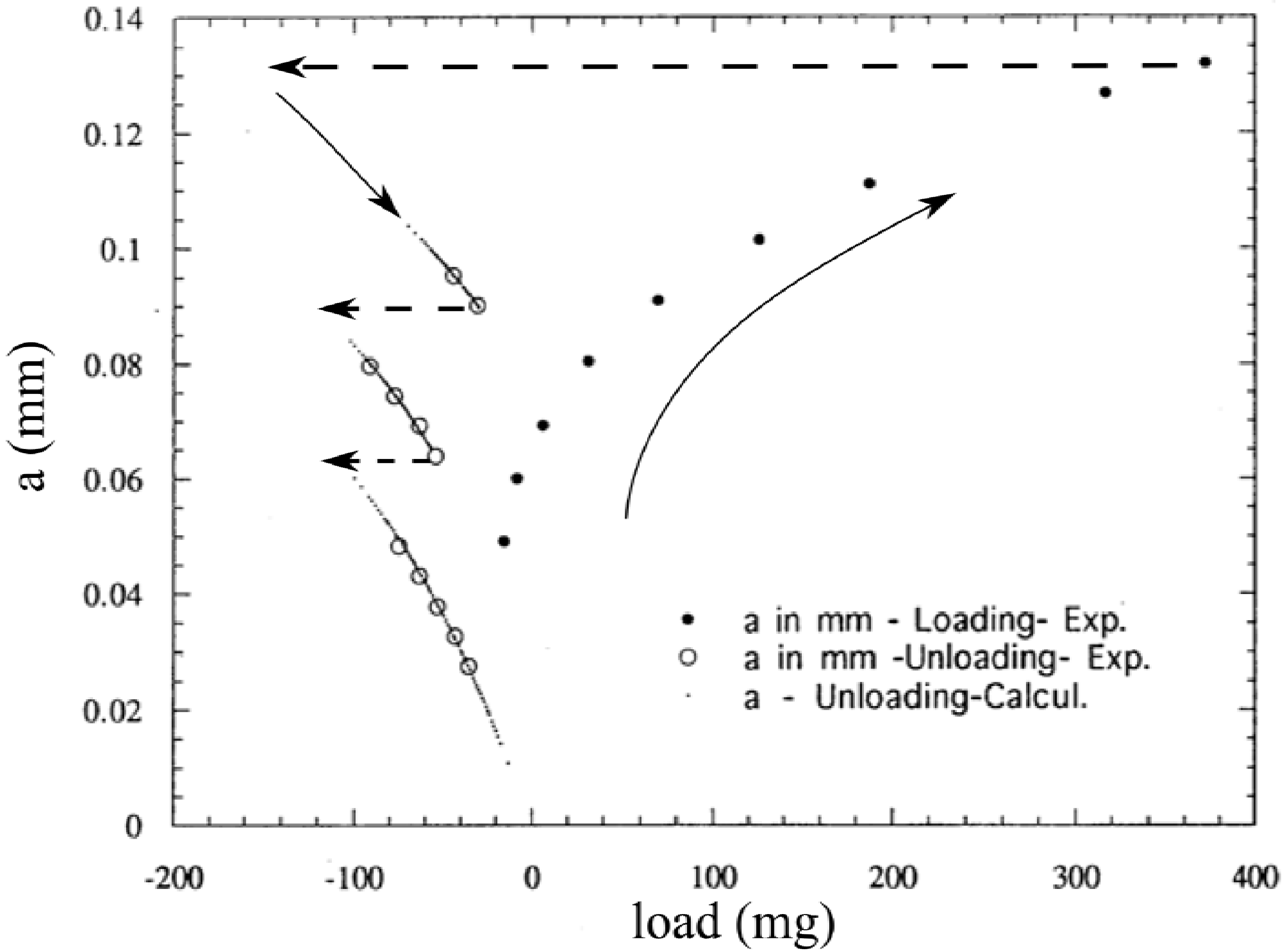
- flat punch displacement

$$2\pi a w = E^{\star}\delta_{fp}^{2}$$

- critical displacement at rupture

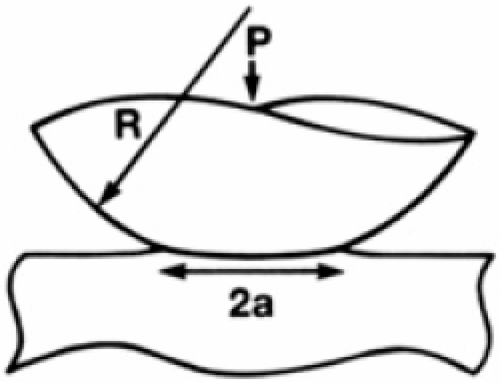
$$\delta_{\mathrm{fp}} \simeq \left(\frac{\pi w^2 R}{E^{\star 2}}\right)^{1/3}$$

Interface characterization by the JKR test

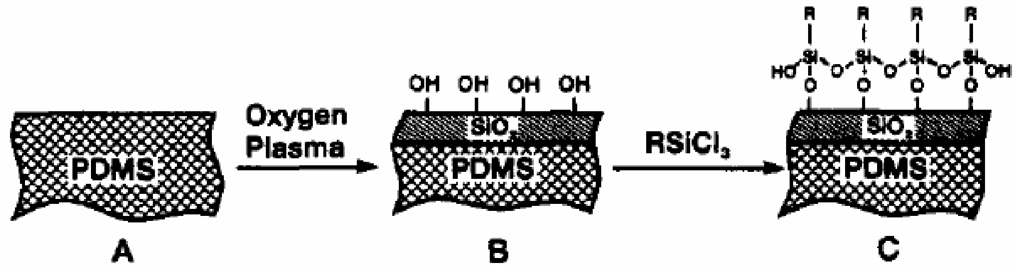


after Deruelle et al. 1995

Adhesion / surface energy measurements



Scheme I. PDMS Functionalized by Oxidation in an Oxygen Plasma To Generate a Silica Surface[a]



[a] This superficial silica layer is further functionalized by reaction with functional alkyltrichlorosilanes.

Table I. Surface Free Energies of Silane-Modified PDMS Surfaces[a]

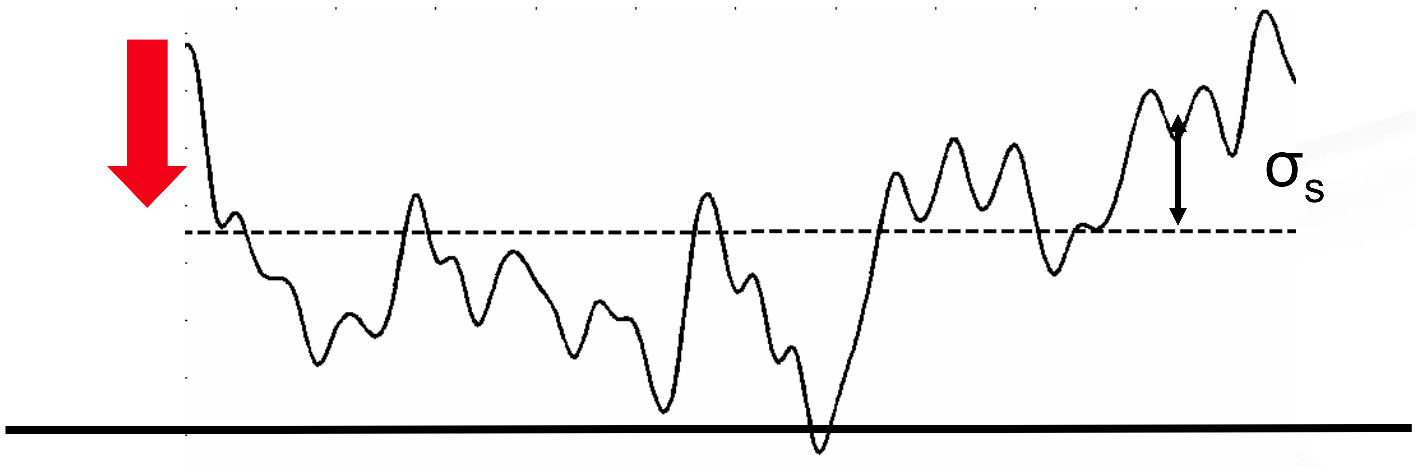
system	θ_a, (deg)	θ_r, (deg)	γ_{sv}, ergs/cm^2, from θ_a	γ_{sv}, ergs/cm^2, from θ_r
PDMS	40 (s)	26 (s)	21.6	24.9
$PDMS^{ox}$	0	0	–	–
$PDMS^{ox}–O_3Si(CH_2)_9CH_3$	42	40	21.0	21.6
$PDMS^{ox}–O_3Si(CH_2)_2(CF_2)_7CF_3$	83	69	8.7	12.8

Chaudhury, Langmuir 7 (1991) 1013

2b) Roughness

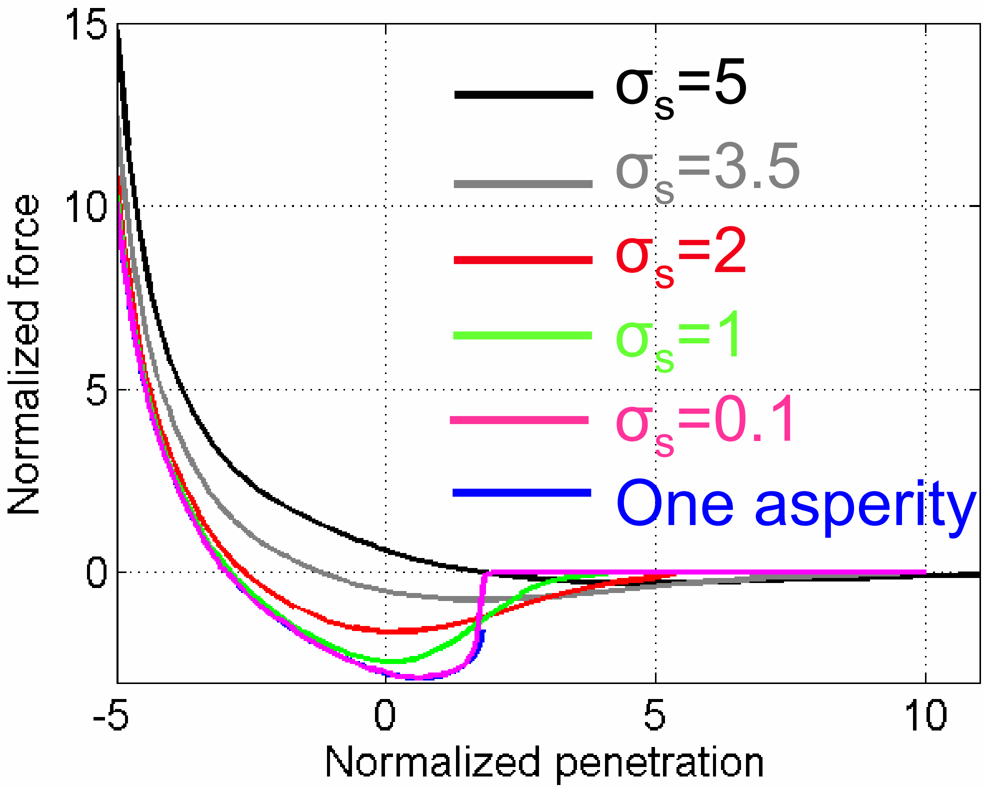
un-correlated identical asperities

Greenwood et Williamson, 1968



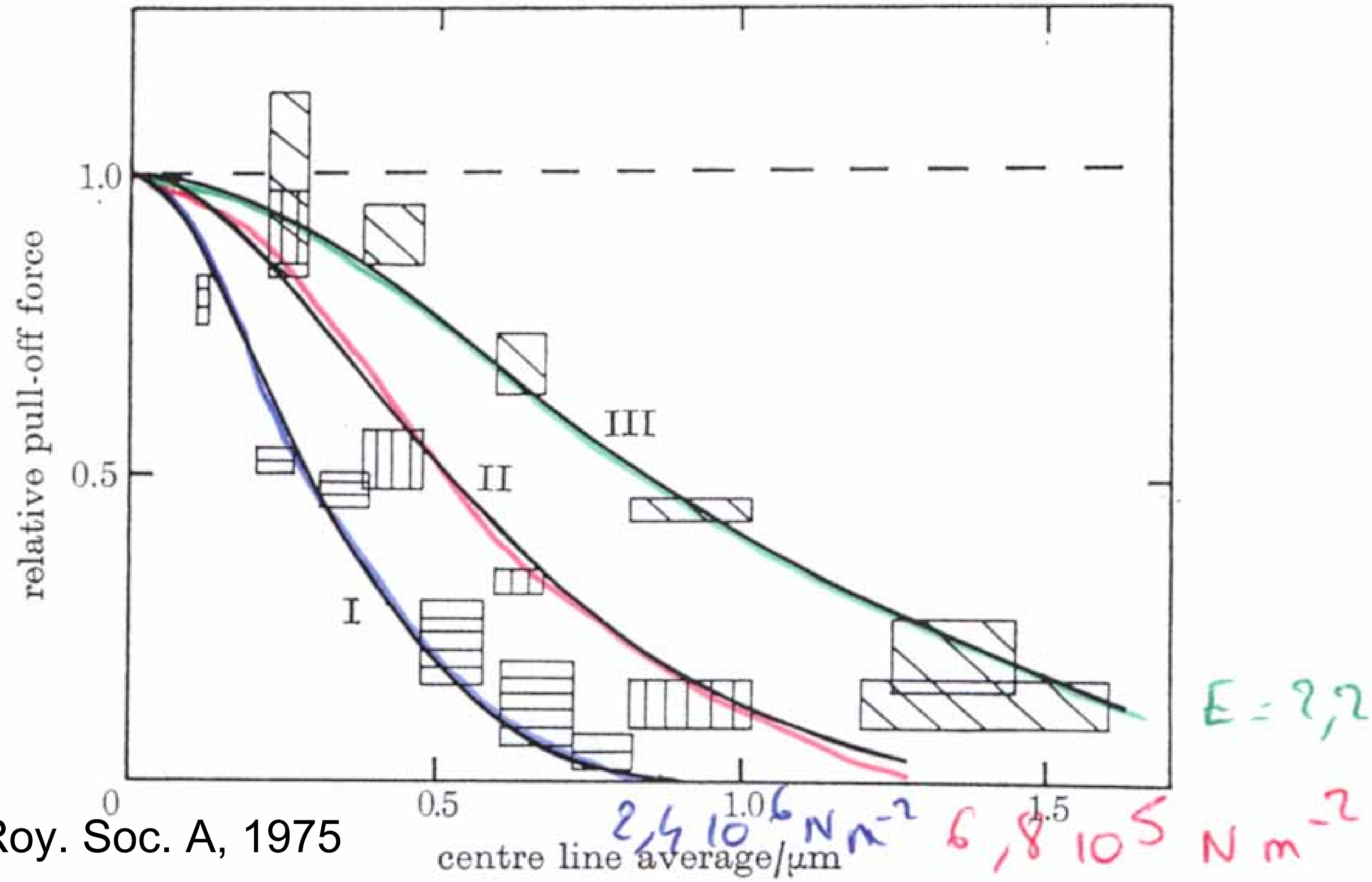
$$\chi(z) = \frac{1}{\sigma_s\sqrt{2\pi}}\exp(-\frac{z^2}{2\sigma_s})$$

Impact of roughness



$$\delta_{\mathrm{fp}} \simeq \left(\frac{\pi w^2 R}{E^{\star 2}}\right)^{1/3}$$

Impact on roughness – elastic adhesive contact



Fuller, Proc. Roy. Soc. A, 1975

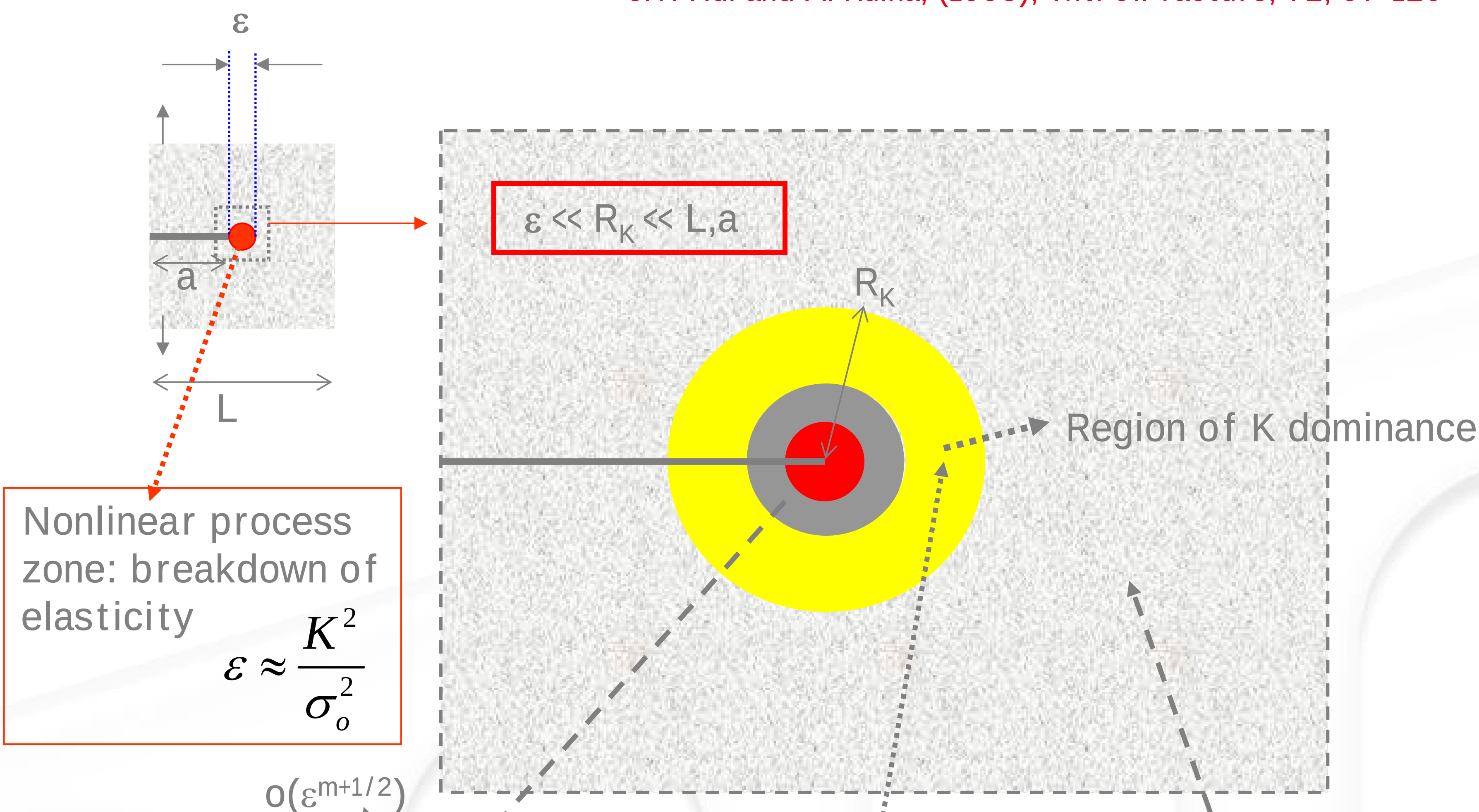
- roughness lowers the “colloidal” cut-off distance

Part IV – interaction stresses, stress intensity factor and remote loading

- 1) The cohesive zone
- 2) Size effects
- 3) Mechanical dissipation and effective adhesion

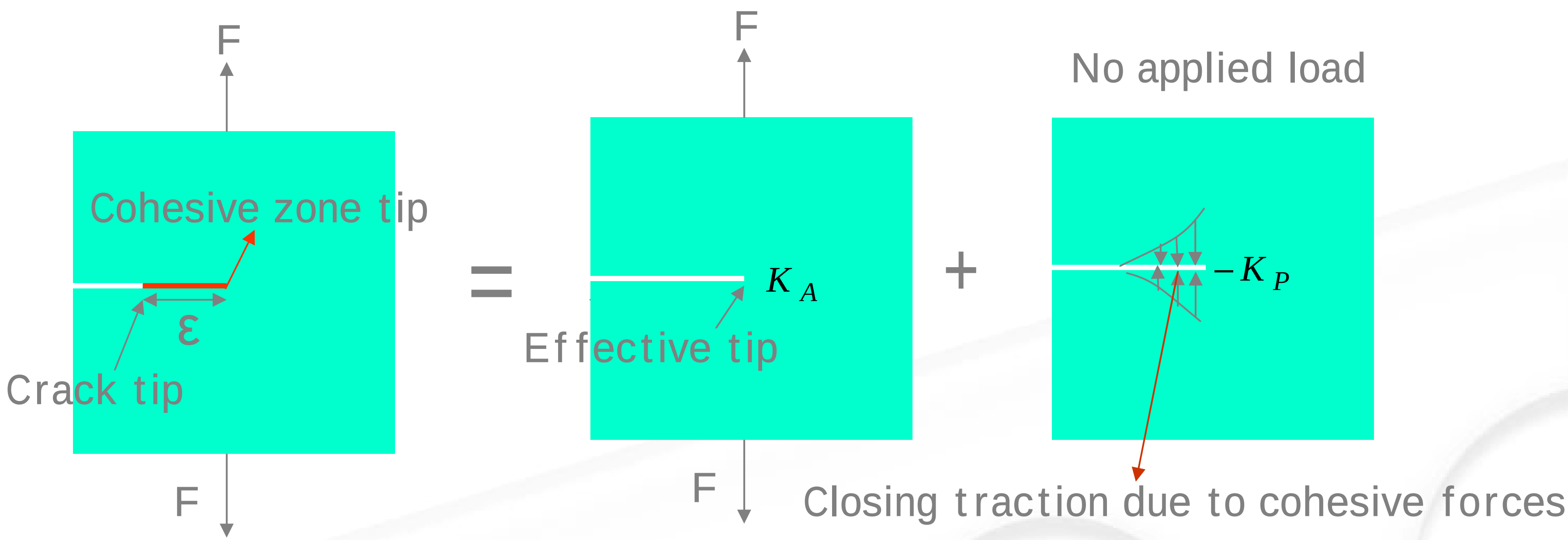
Length scales and Small scale yielding (SSY)

C.Y. Hui and A. Ruina, (1995), Int. J.Fracture, 72, 97-120



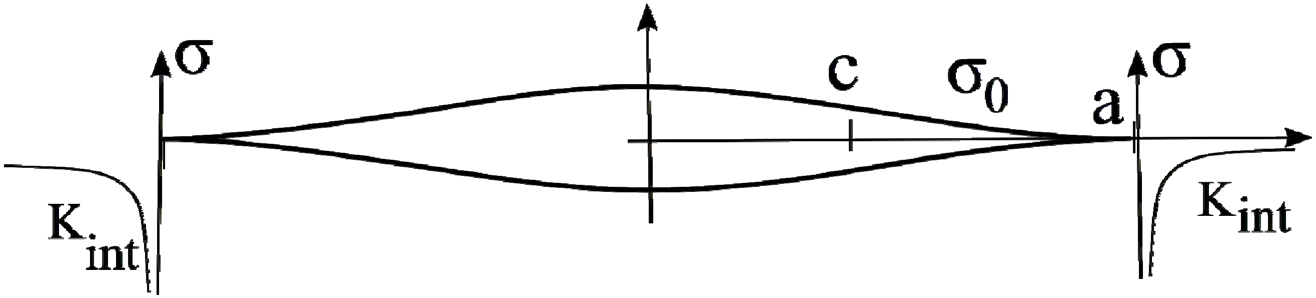
$$\sigma = ...+A_{-m} z^{-(m+1)/2} + ...+ K z^{-1/2} + A_o + ...+ A_j r^{(j+1)/2} + ...$$

1) Cohesive zone



H. Hui

Cohesive zone



- impact of interaction stresses at the crack tip

$$K_{int} = 2\sqrt{\frac{a}{\pi}}\sigma_0 \arccos\left(\frac{c}{a}\right)$$

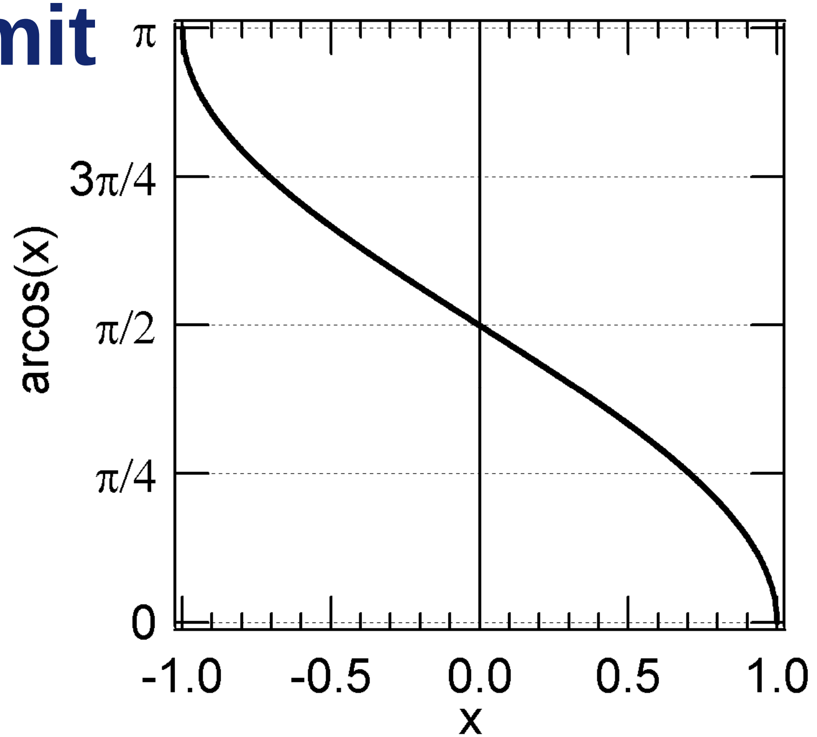
- Full crack

$$c = 0$$

$$K_{int} = \sqrt{\frac{\pi}{2}}\sigma_0\sqrt{a}$$

Maugis, Contact, adhesion and rupture, Springer, 2000, p. 174
Dugdale,D.S., (1960), J. Mech. Phys. Solids, 8,100-104
Barenblatt,G.I., (1962), Advan. Appl.Mech. 7

Cohesive zone – SSY limit



$$\epsilon \equiv a - c, \text{ for } \epsilon/a \ll 1$$

$$\arccos\left(1 - \frac{\epsilon}{a}\right) \simeq \sqrt{2\frac{\epsilon}{a}}$$

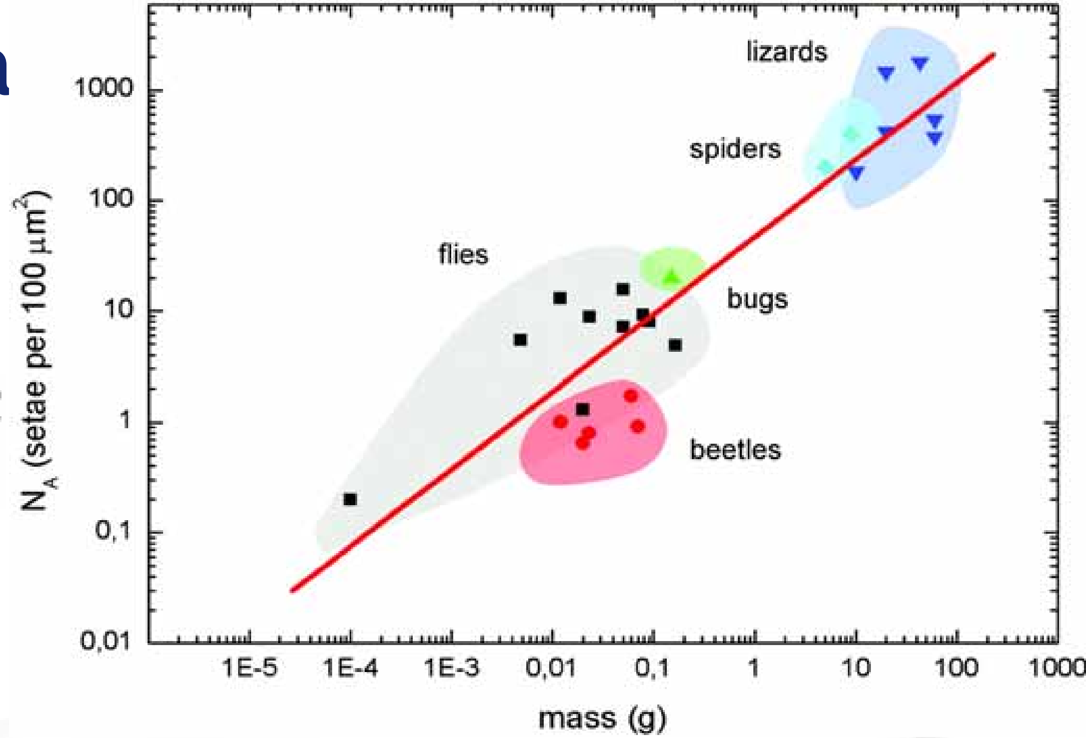
$$\boxed{K_{int} = 2\sqrt{\frac{2}{\pi}}\sigma_0\sqrt{\epsilon}}$$

$$\mathcal{G} = \frac{8}{\pi}\frac{\sigma_0^2\epsilon}{E}$$

2) Adhesive conta

$$2\pi a w = E^\star \delta_{fp}^2$$

$$2aE^\star \delta_{fp} \simeq \sigma \pi a^2$$

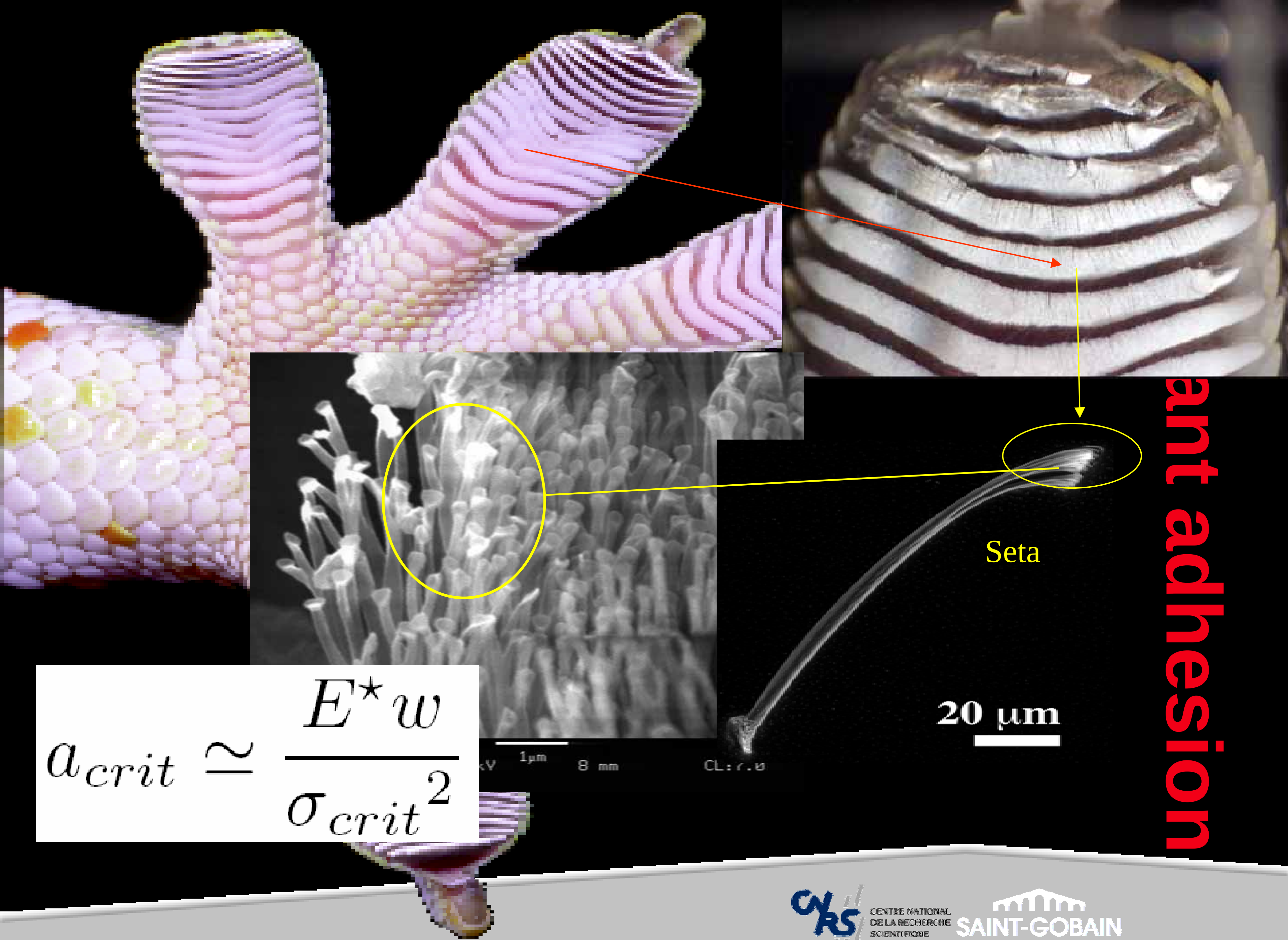


$$\sigma \simeq \sqrt{\frac{wE^\star}{a}}$$

$$a_{crit} \simeq \frac{E^\star w}{{\sigma_{crit}}^2}$$

Gorb and Arzt

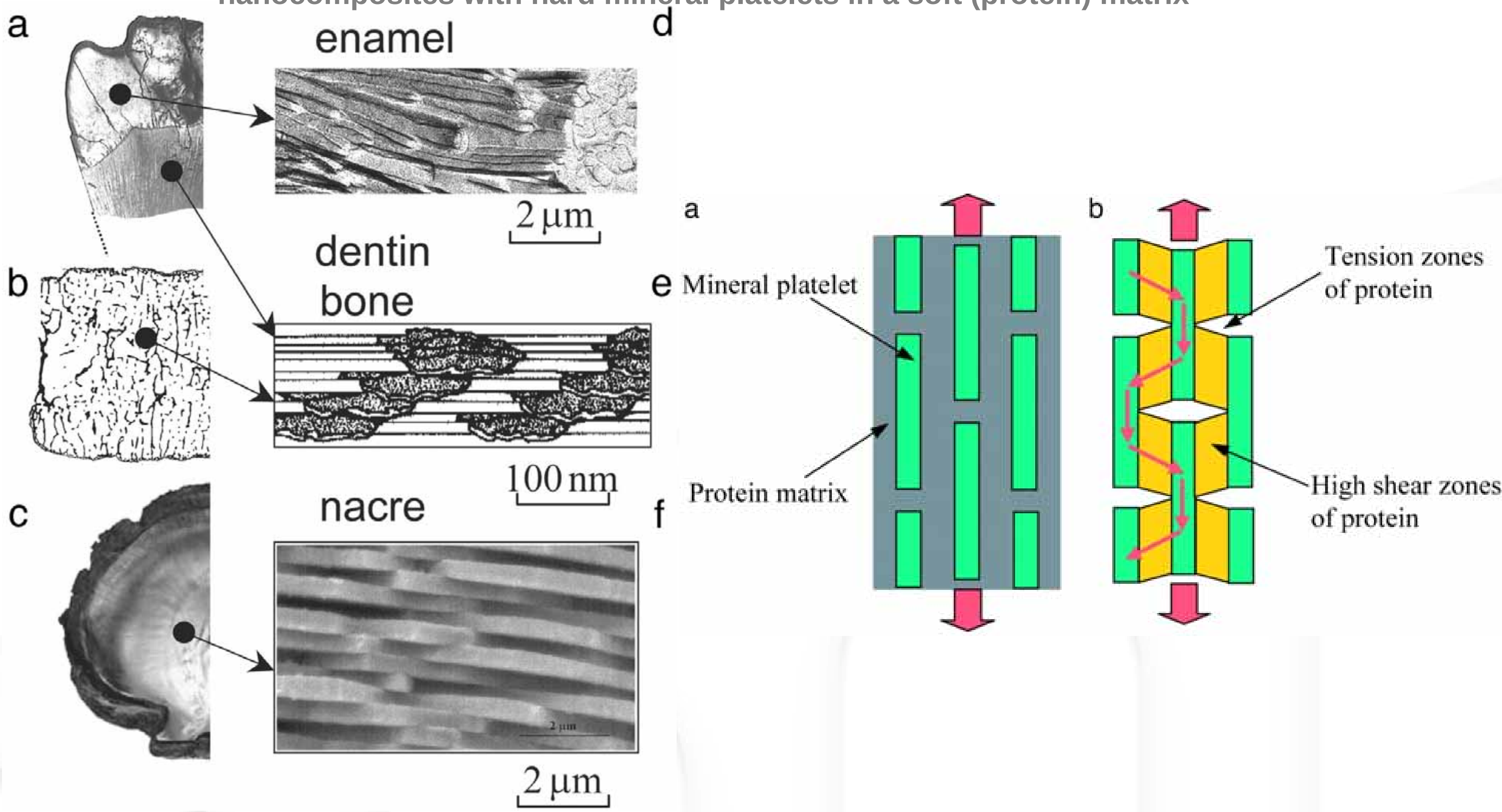
ant adhesion



$$a_{crit} \simeq \frac{E^{\star} w}{{\sigma_{crit}}^2}$$

Flaw tolerant biocomposites

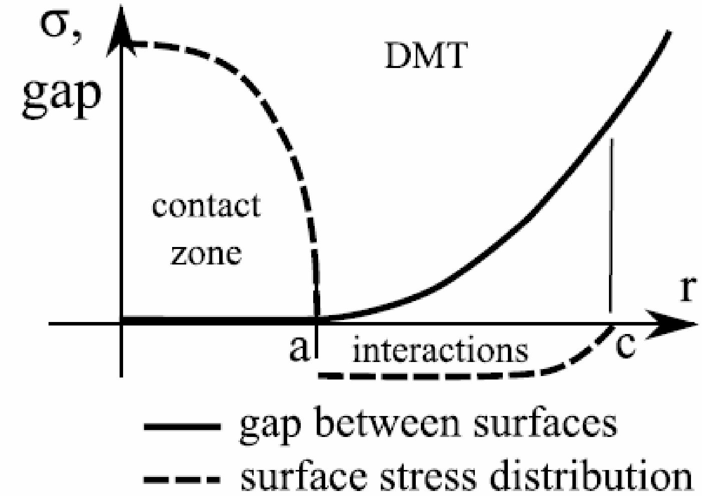
Fig. 1. Many hard biological tissues, such as tooth (a), vertebral bone (b), or shells (c) are made of nanocomposites with hard mineral platelets in a soft (protein) matrix



Gao, Huajian et al. (2003) Proc. Natl. Acad. Sci. USA 100, 5597-5600

3) Mechanical dissipation – effective adhesion

Finite cohesive zone size

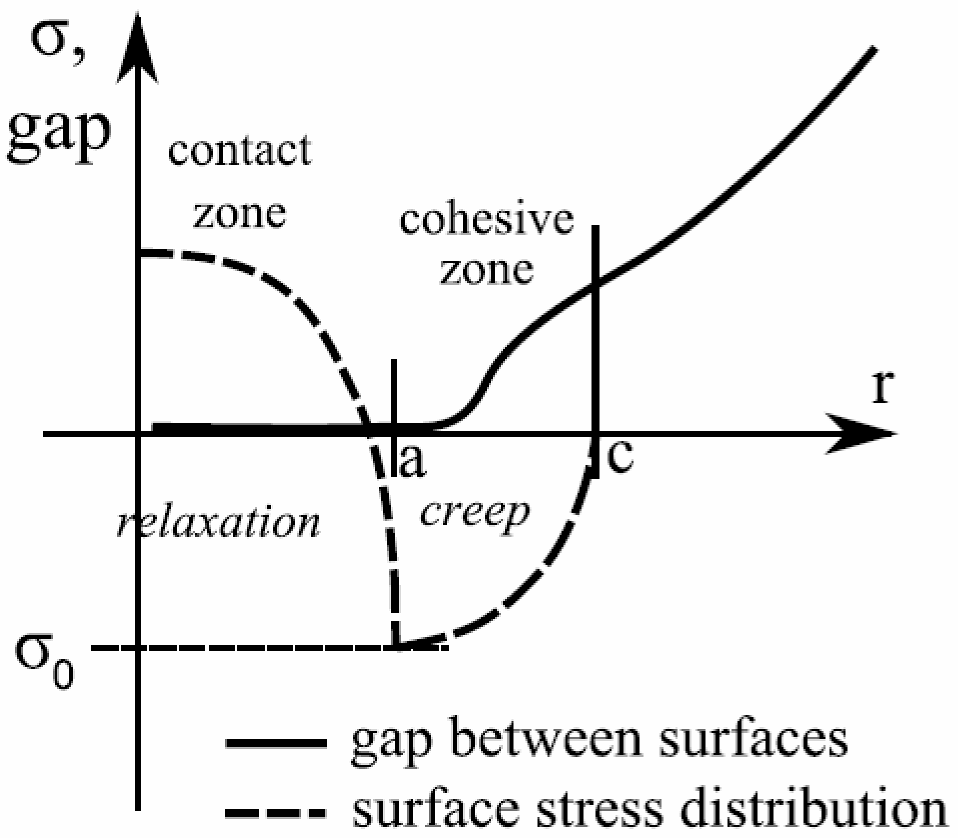


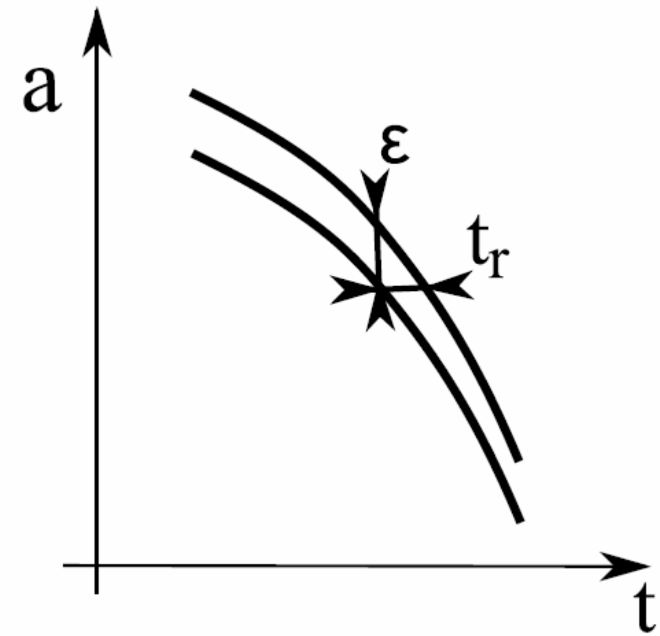
- Cohesive zone size

$$w = \frac{\pi}{4}\frac{{\sigma_0}^2\epsilon_0}{{E_0}^\star}$$

$$\epsilon_0 = \frac{4}{\pi}\frac{w{E^\star}_0}{{\sigma_0}^2}$$

Self-consistent contact description – viscoelastic system

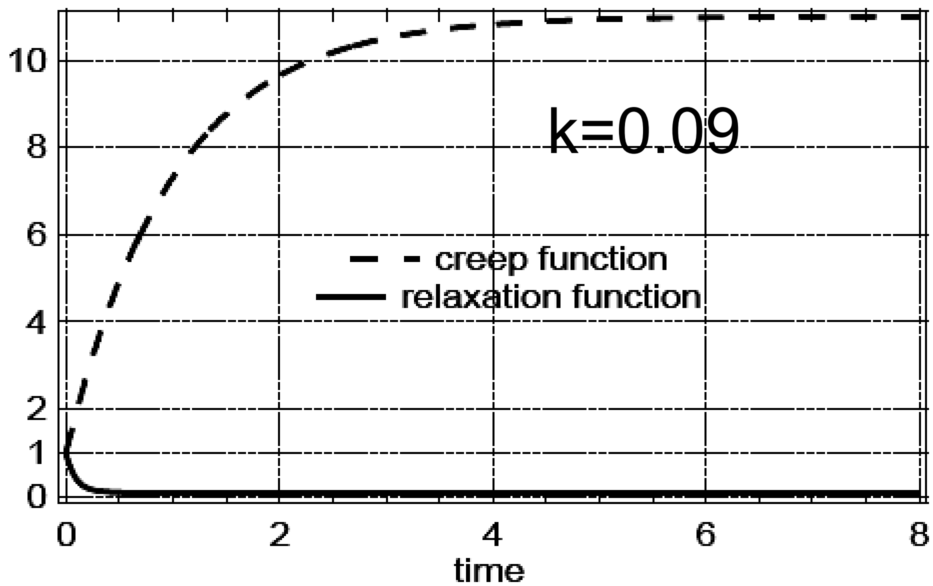




opening

G. Haiat, E. Barthel
JMPS 2003, Langmuir 2002

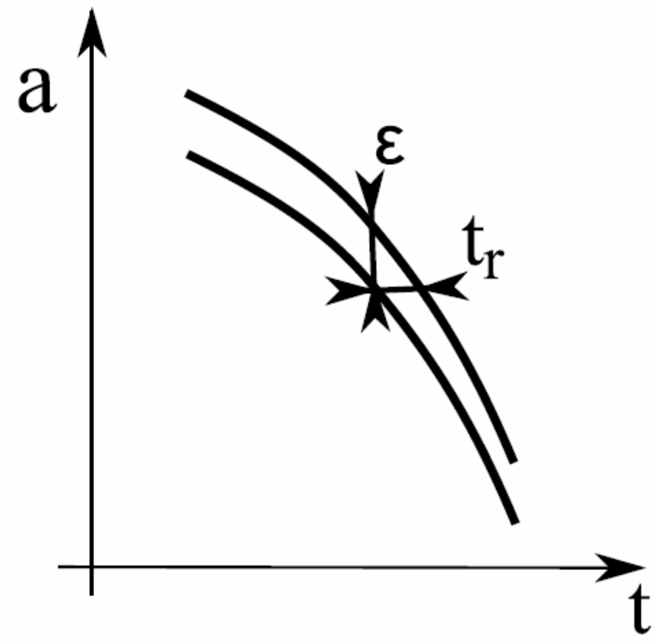
Description of viscoelasticity



$$E_0{}^{\star -1}\tilde{\phi}\left(\frac{t}{T}\right) \equiv \phi(t)$$

$$\tilde{\phi}(t) = 1 + \frac{1-k}{k}\left(1 - \exp(-t)\right)$$

Viscoelastic case – receding contact radius – debonding (opening)

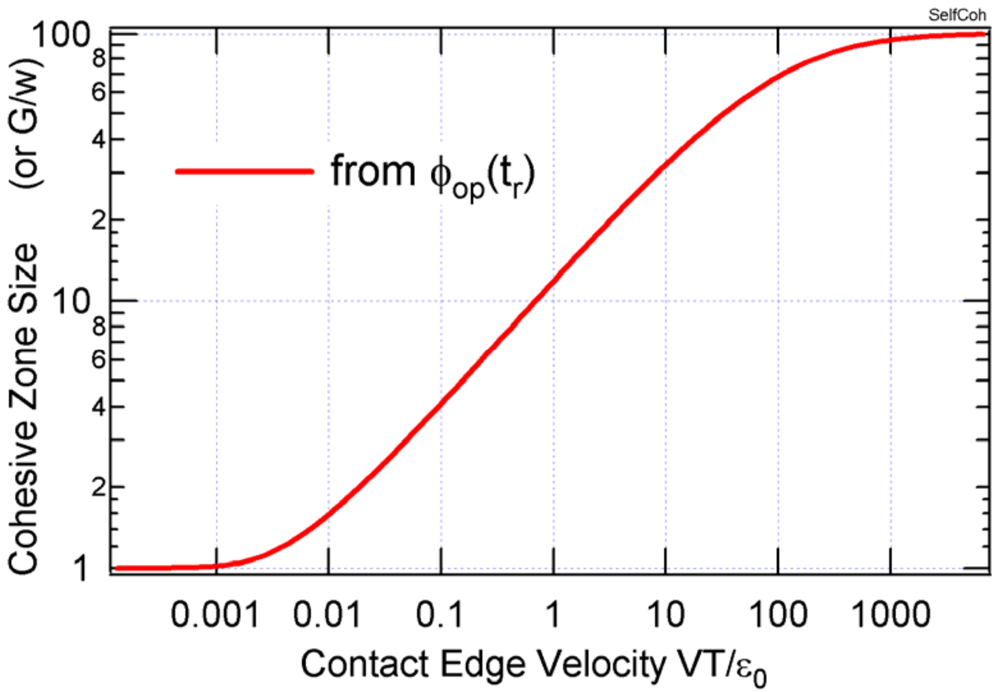


$$\epsilon = \frac{4}{\pi} \frac{w E_0^{\star}}{\sigma_0^2 \tilde{\phi}_{op}(t_r)}$$

$$\tilde{\phi}_{op}(t) = \frac{2}{t^2} \int_0^t \tau \tilde{\phi}(\tau) d\tau$$

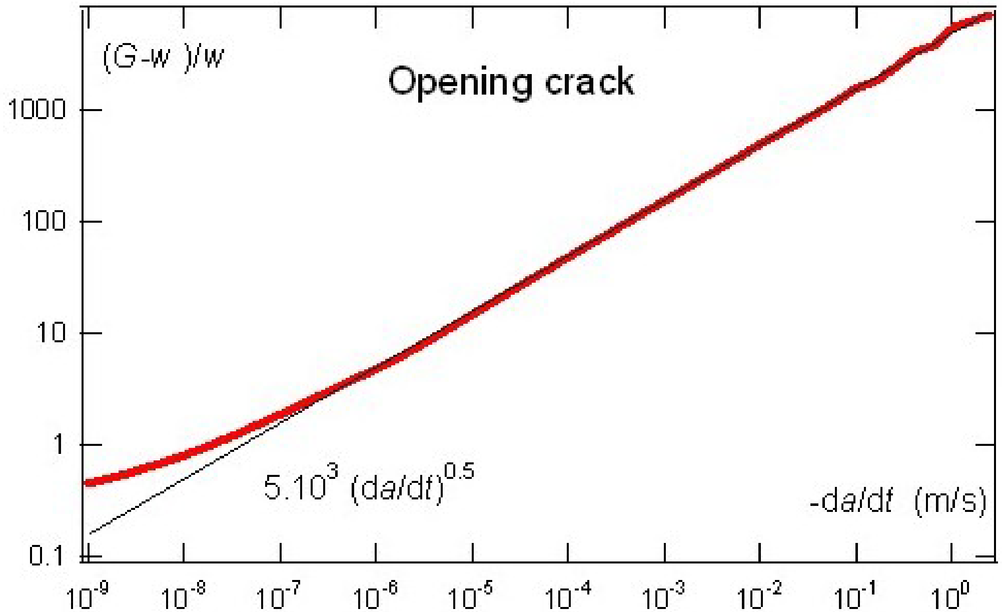
G. Haiat, E. Barthel Langmuir 2002

Viscoelastic adhesive contact – cohesive zone size



$$\epsilon_0 = \frac{4}{\pi} \frac{w E^{\star}{}_0}{\sigma_0{}^2}$$

Viscoelastic dissipation



avec C. Frétigny

Barquens 2006

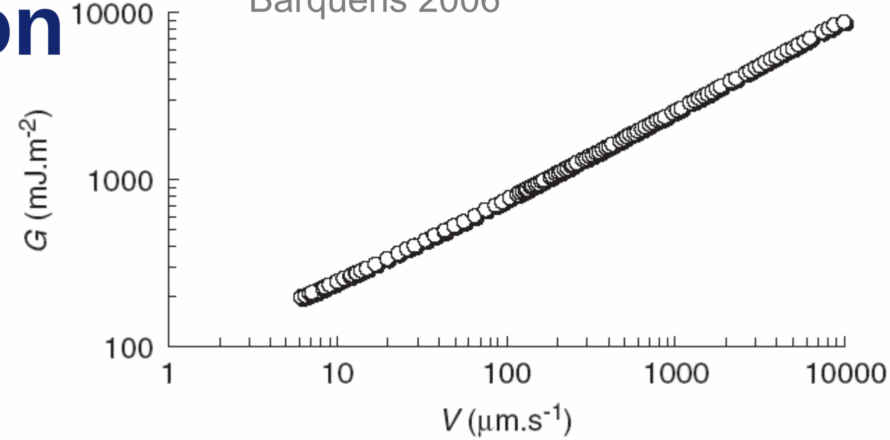


Fig. 5. Strain energy release rate G Versus crack propagation speed V at the interface between the 6 mm diameter glass ball and the rubber surface.

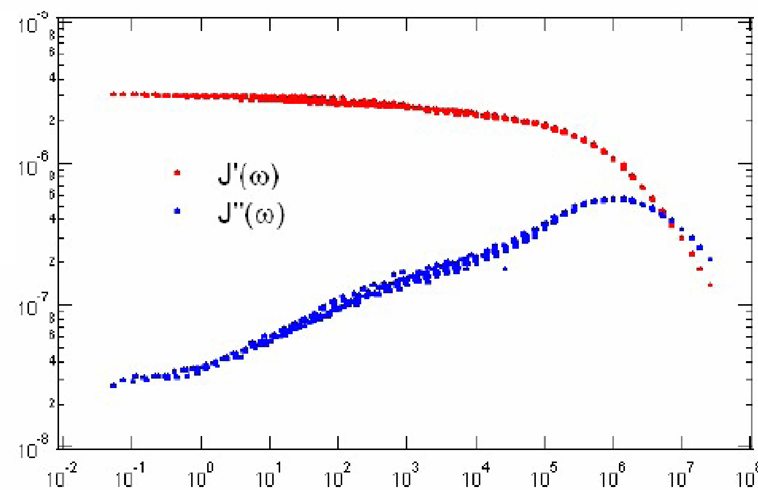


Figure 2: Real and imaginary parts of the compliance of the "'Barquins rubber"'.

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Conclusions

- surface forces and interactions

  self-cleaning glass

  - varied… not only van der Waals
- overall energy balance useful for failure analysis

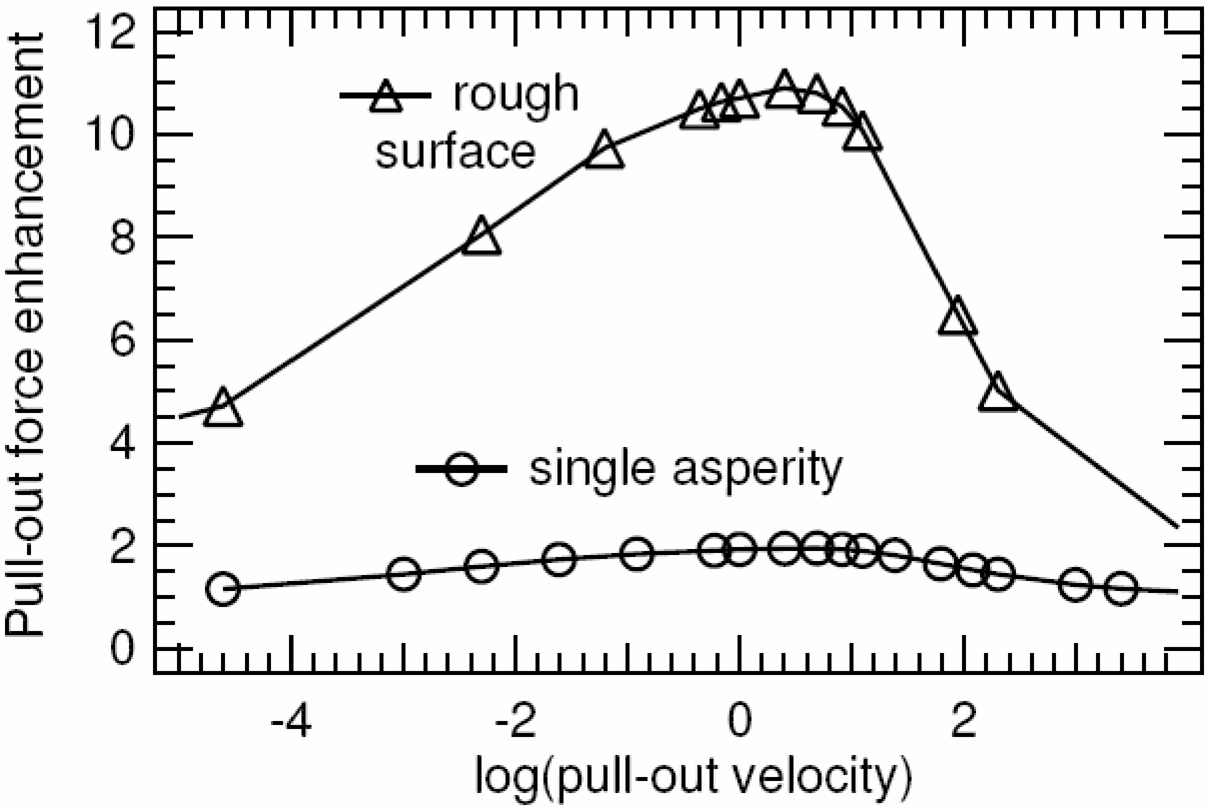
  DCB testing of thin films, crack path selection
- energy transfer between interface and remote loading mediated by stress singularity

  JKR test / impact of roughness on particle adhesion
- cohesion/adhesion stresses control rupture at the crack tip especially through additional dissipative mechanisms with mechanical origin.

  viscoelastic dissipation at the interface: laminated windshields

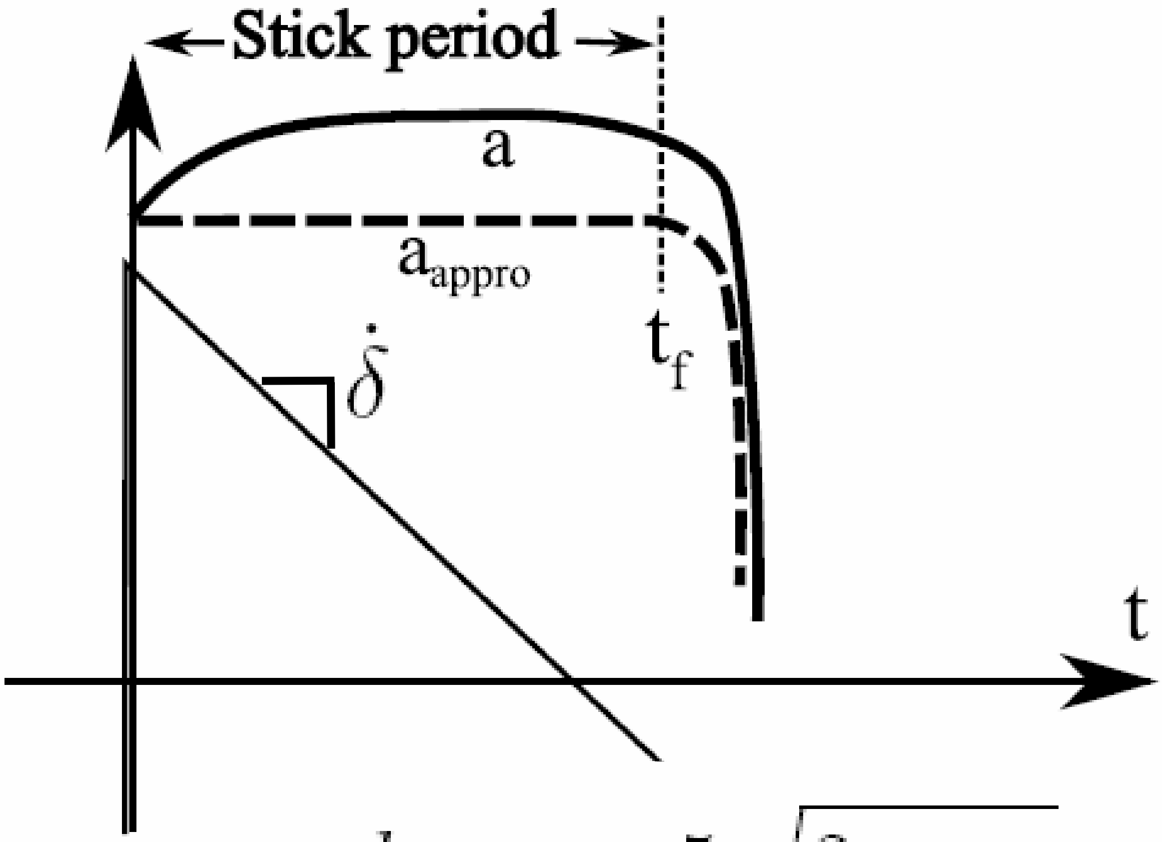
Stick transition

Viscoelastic adhesive contact and roughness



Haiat and Barthel, Langmuir, 2007

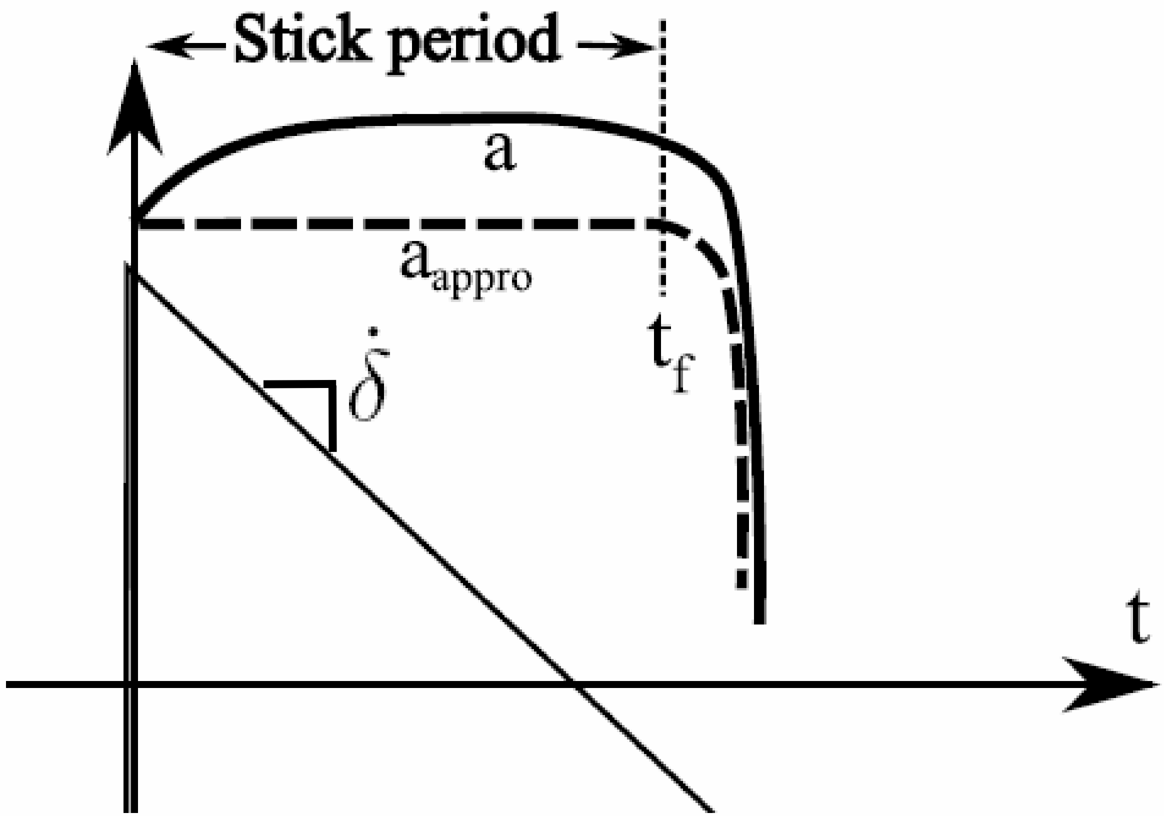
Non reversible contact – stick period



no stick period if:

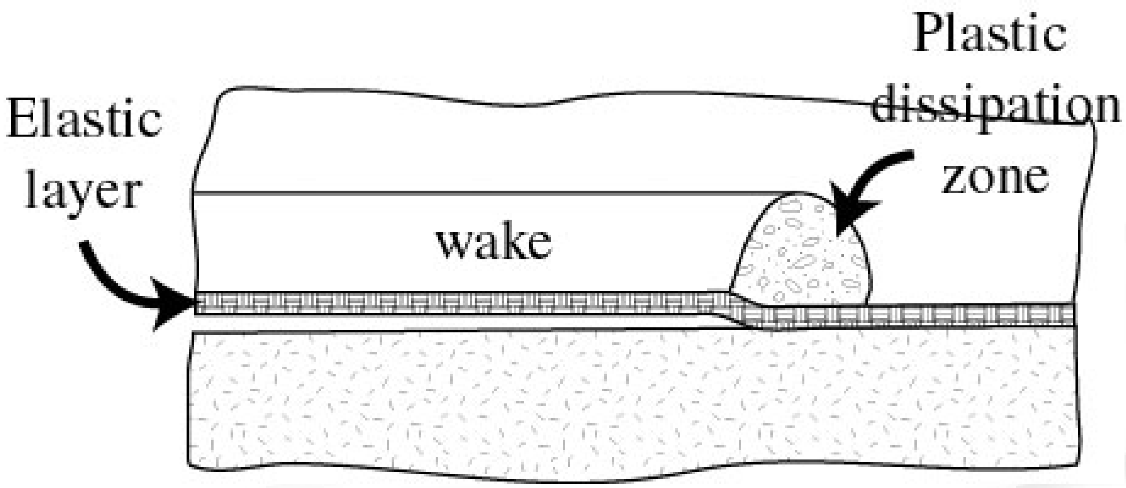
$$|\dot{\delta}| \frac{k\mu}{(1-k)} > \frac{5}{6}\sqrt{\frac{2w\pi a_0}{E^*}}$$

End of stick period – stick transition



$$\dot{\delta} \int_0^{t_f} d\tau \psi(t_f - \tau) + \sqrt{\frac{w \pi a_0}{\phi_{1,r}(t_f)}} = 0$$

Plastic dissipation at the crack tip



Plastic zone size is about 1 μm

adherence

Suo, Shih and Varias, Acta Met. Mat., 1993