Normal contact and friction of rubber with model randomly rough surfaces

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Friction between macroscopic bodies: a longstanding problem....

- **Surface geometry & contact mechanics**
  
  Actual contact area << Apparent contact area

- **Surface physical-chemistry**

  Molecular scale dissipative processes

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*JH Dietrich & BD Kilgore, Technophys 1996*

*Schallamach, 1963*

*Bowden & Tabor, 1950’s*
Rough contacts mechanics

• Non adhesive single asperity elastic contacts  
  Hertz (1881)

• Adhesive contact between smooth surfaces  
  JKR & DMT (1971)

• Contact between nominally flat surfaces  
  Greenwood & Williamson (1966)

  \[ A \propto P \]

  \rightarrow \text{Justification of Amontons-Coulomb's friction law}
  \rightarrow \text{Extensions to more complex geometries}
  \hspace{1cm} \text{Archard (1957), Ciavarella (2008)…}

• Rough contact models based on a spectral description of surface topography  

  \rightarrow \text{Extensions to friction and adhesive contacts}
Transparent randomly rough surfaces consisting in distributions of spherical asperities (~50 µm)

**GW type surfaces**

Imaging of micro-contacts distributions

Load dependence of the real contact area $A(P)$

Statistical distributions of micro-contacts pressure and size

**Role of elastic coupling between asperities?**

Without elastic coupling

With elastic coupling

**GW model**
Patterned surfaces and associated sphere-on-plane contacts / I

- **Soft Asperities (SA) surfaces**

  Smooth glass lens ($R=128\ \text{mm}$)

  Patterned elastic PDMS substrate

  Replication of a micro-machined PMMA template

  - Lateral and height distributions of spherical asperities perfectly controlled by design
    
    $\rightarrow$ Uniform random height distribution ($R=100 \ \mu\text{m}, \ 30 \ \mu\text{m} < \text{height} < 60 \ \mu\text{m}$)

  - Small scale roughness on the micro-asperities $\rightarrow$ normal contact experiments only

        Surface density: \( \phi = 0.1 \) and 0.4
Patterned surfaces and associated sphere-on-plane contacts /II

- **Rigid Asperities (RA) surfaces**

  Patterned glass lens ($R=13 \text{ mm}$)

  Surface density: $\phi=0.41$

  smooth PDMS substrate

- Gaussian distribution of asperity sizes and heights (a posteriori characterization)

- Smooth micro-asperities $\rightarrow$ normal contact and friction experiments

Water droplet condensation method....
Fabrication of rigid asperities patterns by droplet condensation method

1. Water droplet condensation
   - Silanized glass (HMDS)
   - $H_2O$, $T^\circ$

2. PDMS replica
   - Sylgard 184 + crosslinker

3. Sol gel replica on a glass lens
   - PDMS mould
   - Glass lens
   - Reactive sol-gel solution

Time of exposure to water vapor
- Size of the droplets

Shape of the asperities controlled by the contact angle

$h = R(1 - \cos \theta)$

$\theta \sim 57^\circ$

![Graph showing the relationship between height and radius](image)
Contact devices

Contact imaging of micro-asperities contacts

✓ Contact radii & spatial distributions of micro-contacts (RA & SA surfaces)

✓ Contact pressure distribution (RA surfaces only)

Hertz law assumed to be obeyed locally

\[
P_i = \frac{16a_i^3}{9R_i}
\]
Load dependence of the real contact area $A(P)$

Power law dependence

$$A \propto P^n$$

RA surface $n=0.81 \pm 0.01$

SA surface $n=0.94 \pm 0.01$ independent of surface density of micro-asperities
A(P) relationship: role of elastic interactions?

- Modified form of the GW model: Ciavarella’s model
  
  Ciavarella, 2008

Indentation depth of the $i^{th}$ micro-asperity contact:

$$\delta_i = \delta_i^0 + \sum_{j \neq i}^{N} \alpha_{ij} \delta_j^{3/2},$$

Geometrical term \hspace{2cm} Elastic interaction term

→ With elastic interactions

$$[\alpha_{ij}] = -\frac{4\sqrt{R_j}}{3\pi} \frac{1}{r_{ij}}, \quad i \neq j,$$

→ Without elastic interactions

$$[\alpha_{ij}] = 0$$
Calculated load dependence of the real contact area

With elastic interactions $[\alpha_{ij}] \neq 0$

Without elastic interaction $[\alpha_{ij}] = 0$
Departure of the $A(P)$ relationship from linearity

- Lens curvature effect

$A \propto P^n$

RA surface $R_l = 13$ mm $n = 0.81$
SA surface $R_l = 128$ mm $n = 0.94$

Gap between the nominal ~ micro-asperity size $\rightarrow \zeta \propto P^{-1/9}R_l^{5/9}$
Micro-contacts spatial distributions

- SA Surface

$P=0.02 \text{ N}$  \hspace{50pt}  $P=0.2 \text{ N}$  \hspace{50pt}  $P=0.5 \text{ N}$

Predictions from Ciavarella’s model

- $[\alpha_{ij}] \neq 0$
- $[\alpha_{ij}] = 0$
Contact pressure distribution $p(r)$

- **Experimentally**: summation of the local micro-contacts forces $p_i$ within $r$ and $r+dr$ (averaging over more than 20 realizations of the SA contacts)

- **Theoretically**:
  - Ciavarella’s model
  - **Extension of the GW model to the contact of rough spheres**
    Greenwood and Tripp (1967)

- **GT model**

  - No short range elastic interaction between neighboring micro-asperities
  - Long range elastic coupling coming from the curvature of the nominal surfaces
• Added tail to the Hertzian pressure distribution \( \zeta \propto R^{5/9} \sigma^{2/3} \sim \sqrt{R} \sigma \)

No significant difference between Ciavarella’s and GT model

Short range elastic interactions does not affect the radial pressure distribution

What about the distribution of quantities from which \( p(r) \) derives?
Micro-contacts density and average micro-contact radius

**Micro-contact density**

**Average micro-contact radius**

GW model for uniform random height distribution

\[ \eta \propto p^{2/5} \]

\[ \bar{a} \propto p^{1/5} \]

GW model obeyed over most of the contact pressure range
Frictional properties of RA surfaces

\[ \bar{R} = 64.4 \, \mu m \]  \hspace{1cm} V = 0.5 \, \text{mm/s} \]

\[ \bar{R} = 49.6 \, \mu m \]

\[ \tau_0 = 0.34 \, \text{MPa} \]  \hspace{1cm} \tau_0 = 0.49 \, \text{MPa} \]

\[ \tau_0 = 0.40 \, \text{MPa} \]

Smooth lens: \[ F = \tau_0 A \]

Patterned lens: \[ F \neq \tau_0 A \]

\[ A = \sum_i (\pi a_i^2) \]

Interface shear stress cannot simply be transposed at all length scales.
Conclusion / Outlook

- **Normal contact** of model randomly rough surfaces reminiscent to GW model
  - Long range elastic interactions coming from the curved profile of the indenter
  - Short range interactions between neighboring micro-contacts negligible

  Experimental validation of the GW Williamson model

  Extension to more realistic surface roughness including fractal surfaces ??

  → Experiments with hierarchical surface roughness

- Preliminary **friction** results show that frictional stress measured at macroscopic length scales cannot simply be transposed to multi-contact interfaces

  → Dependence of rubber friction on surface stretching ??