Rupture, Fracture and size issues

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SIMM/ESPCI

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Fracture...

Marcel Duchamp, the Large Glass (Philadelphia)

Kendell Geers, Stripped Bare (exhibition in Tours, 2012)

Outline

Theoretical strength

Stress concentration

Energy flow

Fracture and dissipation

Main notions I

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Main notions II

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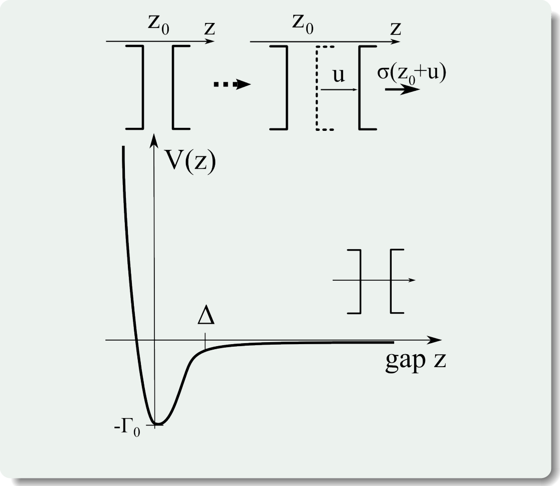
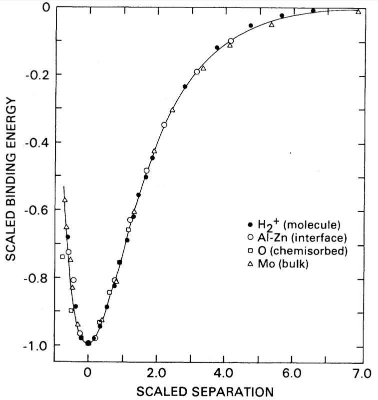
Interaction energy as a function of surface separation



Lawn and Wilshaw (1975)

- interaction potential $V(z)$
- Surface stress $\sigma(z) = -\frac{dV}{dz}$
- **cohesion energy** Γ_0

Normalized interaction potential



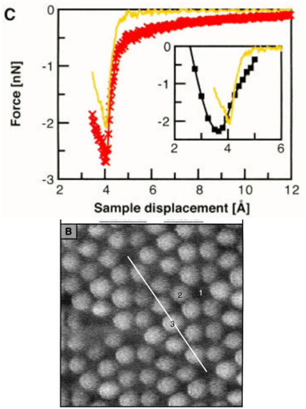
Ferrante et al. (1983)

- $V(z) = \Gamma_0 \tilde{V}(\tilde{z})$
- $\tilde{z} = \frac{z - z_0}{\Delta}$ where
- Δ is defined by $\Gamma_0 = \Delta^2 \left. \frac{d^2 V}{dz^2} \right|_{z_0}$

Can we measure the interaction directly ?

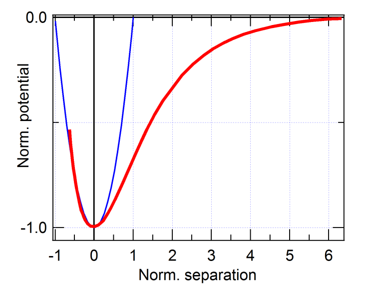
1. Surface forces measurements with fine tips allow for direct measurement of local inter-surface interactions
2. note long range contribution

Lantz et al. (2001)



Tip/surface interaction.

Elastic modulus

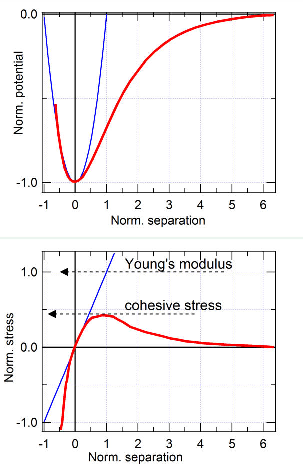


Normalized interaction energy as a function of normalized surface separation

After Ferrante et al. (1983)

- near z_0, $\sigma(z) = -\left.\frac{d^2V}{dz^2}\right|_{z_0}(z - z_0) = E\frac{z-z_0}{z_0}$
- the **elastic modulus** is $E = -z_0 \left.\frac{d^2V}{dz^2}\right|_{z_0} = \frac{z_0}{\Delta^2}\Gamma_0$

Cohesive stress



Surface stress as a function of surface separation

- the **cohesive stress** σ_{coh} is the stress maximum
- $\sigma_{coh} = \frac{\Gamma_0}{\delta}$ where $\delta = 3$ to 8Δ

$$\boxed{\sigma_{coh} \simeq E/8 \text{ to } E/3}$$

Theoretical strength *vs.*. . .

Evaluation of the order of magnitude of the cohesive strength (also called **theoretical strength**)

Order of magnitudes

$$\begin{aligned} \Gamma_0 &\simeq 1\ \mathrm{Jm}^{-2} \\ \Delta &\simeq 0.2\ \mathrm{nm} \\ E &\simeq 100\ \mathrm{GPa} \\ \sigma_{coh} &\simeq 30\ \mathrm{GPa} \end{aligned}$$

which is 10^6 N or

100 tons on 1×1 cm^2 !!!

. . . practical strength !

The recommended loading for architectural glass products are in the range of 10s of MPa...
It is somewhat better but still quite limited for other materials such as metals.

How do I get from the **practical strength** to the stress level needed for material rupture ?

"Antiplane" elasticity

100 % pure shear – same quality, lower price...

Elastic fields and equilibrium

- deformation and stress

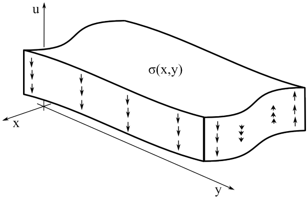
$$\bar{\epsilon} = \nabla u(x, y)$$

$$\bar{\sigma} = \mu \bar{\epsilon}$$

- equilibrium

$$\mathrm{div}(\bar{\sigma}) = 2\mu \triangle(u)$$

$$\boxed{\Delta u = 0}$$



Deformation for antiplane elasticity

One (scalar) field, one single elastic constant.
cf electrostatics, liquid flow...

Fracture – boundary conditions

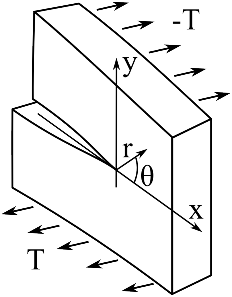
A **fracture** is a free surface with a boundary.

Boundary conditions

- stress

$$\boxed{\sigma_y = 0} \quad \text{for} \quad \theta = \pm\pi$$

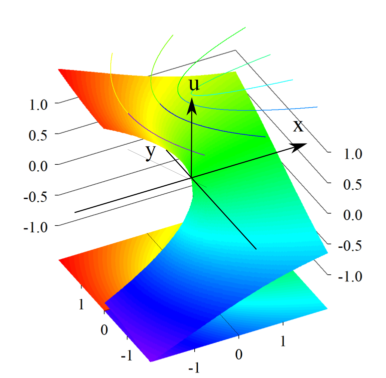
- u is discontinuous on the fracture faces



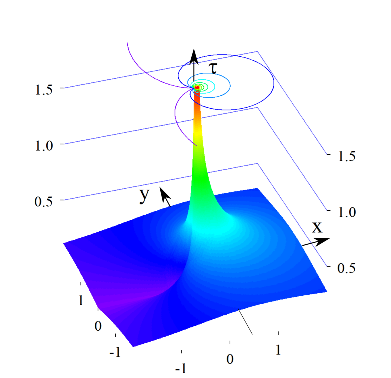
Fracture geometry in mode III

The fracture problem - around the crack tip

The displacement field



The stress field

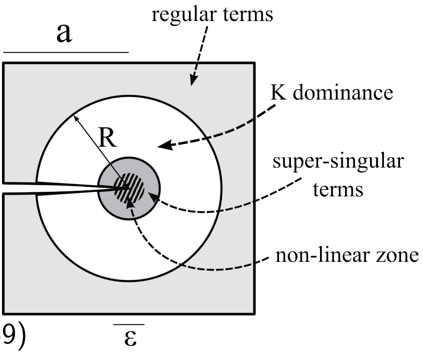


Crack tip stress field

Expansion around the crack tip

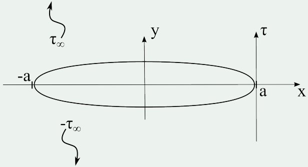
$$\begin{aligned}\sigma = \ldots \quad & + \quad A_{-m} z^{-m-\frac{1}{2}} + \ldots \\ & + \quad K z^{-\frac{1}{2}} \\ & + \quad A_0 + \ldots + A_m z^{m+\frac{1}{2}} + \ldots\end{aligned}$$

Williams expansion – Williams (1959)



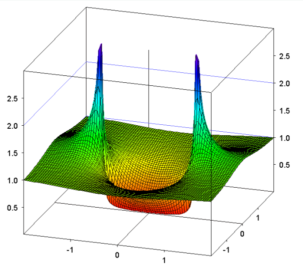
The area of K dominance.

A perfect 2D crack with far field stresses τ_∞ – the **slit crack**



$$\tau(x,0^+) = \frac{\tau_\infty x}{\sqrt{x^2 - a^2}}$$

$$\frac{x}{\sqrt{x^2-a^2}} = \frac{\sqrt{a}}{\sqrt{2}}\frac{1}{\sqrt{x-a}} + \frac{3}{4\sqrt{2}\sqrt{a}}\sqrt{x-a} + \frac{5}{32\sqrt{2}a^{3/2}}(x-a)^{3/2} + \cdots$$



Stress field distribution τ

For a "perfect" crack, the *most singular* term is the $\sigma \propto \frac{1}{\sqrt{r}}$ term.

A connection between far field and crack tip stress field

The **stress intensity factor** K is defined by

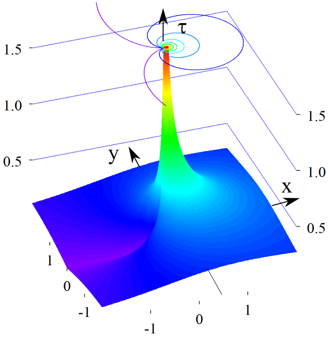
$$\sigma(r) \simeq \frac{K}{\sqrt{2\pi r}}$$

For our 2D case

$$\tau(r,\theta) \simeq \tau_\infty \sqrt{\frac{a}{2}} \frac{1}{\sqrt{r}} \left(-\sin\frac{\theta}{2}, \cos\frac{\theta}{2}\right)$$

so that

$$\boxed{K = \frac{1}{\sqrt{\pi}}\, \tau_\infty \sqrt{a}}$$



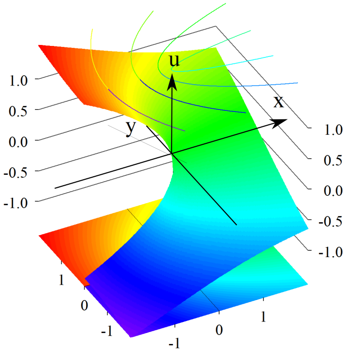
Stress distribution around the crack tip.

Note: the angular dependance shown here is specific to this special case.

Crack tip – 2D elasticity

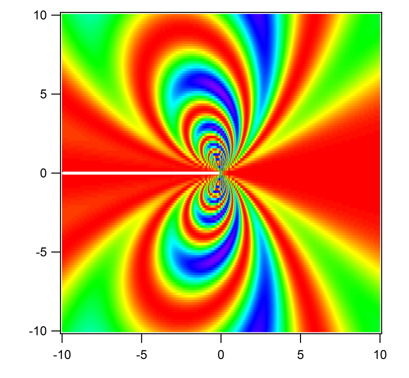
displacement field

$$u(r,\theta) = \frac{K}{\mu}\sqrt{\frac{2r}{\pi}}\sin\frac{\theta}{2}$$

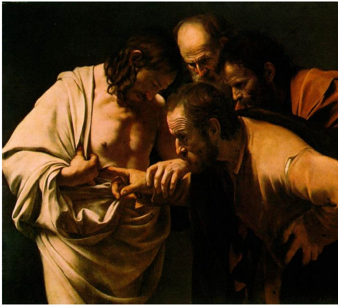


Displacement distribution around the crack tip.

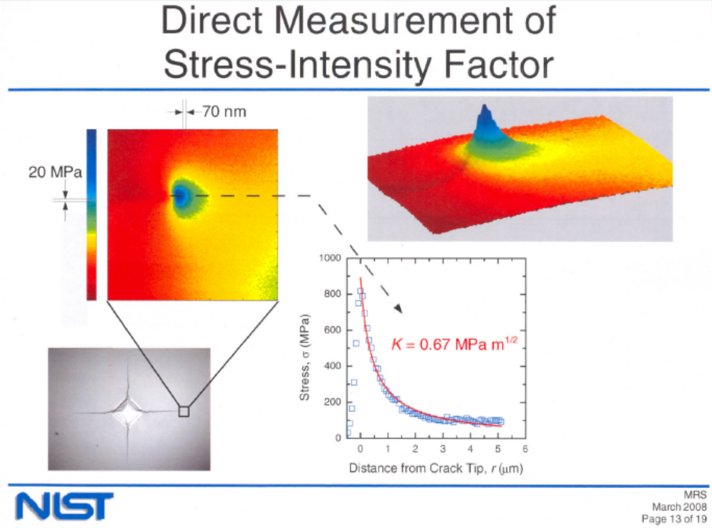
Seeing the K field



A typical photoelastic stress field around a crack tip.

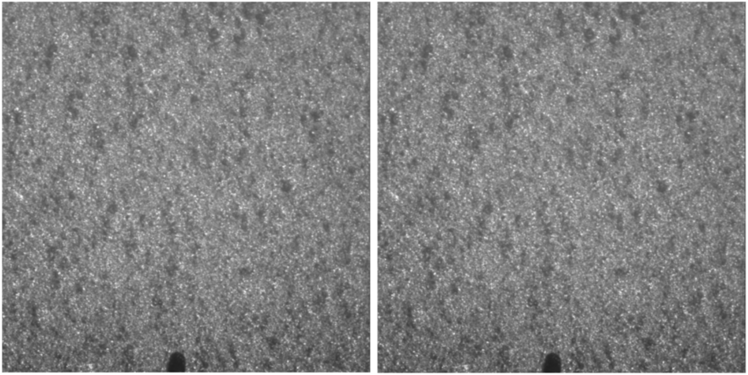


With the correct angular dependance for a real, mode I crack.



Measured crack tip stress field. After Cook 2008

Where is the crack ?



Two successive images of a propagating crack – SiC.

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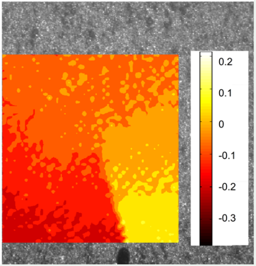
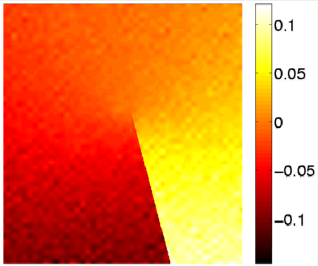
Galerkin approach to **Digital Image Correlation**



Q_4 elements1 px $= 1.85\ \mu$m



XQ_4 (extended) elements, with a discontinuity field – 1 px $= 1.85\ \mu$m

$\Phi^2(a_i) = \int\int (g(\bar{x}) - f(\bar{x}+\bar{u}))\, d\bar{x}$ with $\bar{u}(\bar{x}) = a_i \phi_i(\bar{x})$

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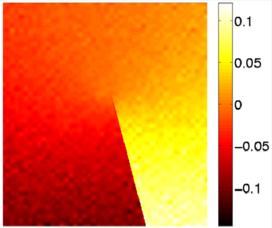
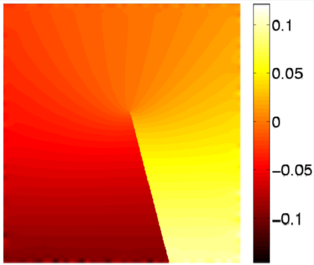
Extended integrated elements



XQ_4 (extended) elements, with a discontinuity field – 1 px = 1.85 μm



XIQ_4 (extended and integrated) with a Williams expansion

similar to

$$\bar{u} = \ldots + A_{-m} z^{-m+\frac{1}{2}} + \ldots + K z^{\frac{1}{2}} + A_0 z + \ldots + A_m z^{m+\frac{3}{2}} + \ldots$$

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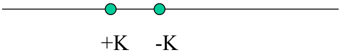
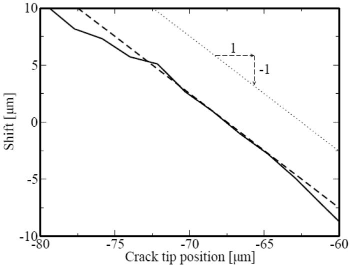
What are the supersingular fields useful for ? (-1)



Crack dipole



Shift as a function of position

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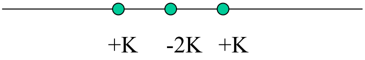
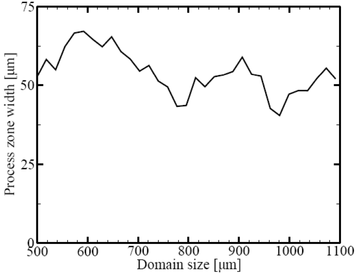
What are the supersingular fields useful for ? (-2)



Crack quadrupole



Plastic zone size

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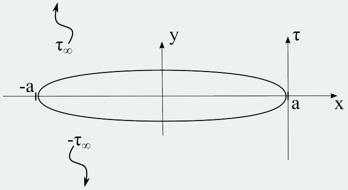
If the stress distribution is singular, what happens when it goes to infinity ?

A perfect 2D crack with far field stresses τ_∞
elastic shear modulus: μ



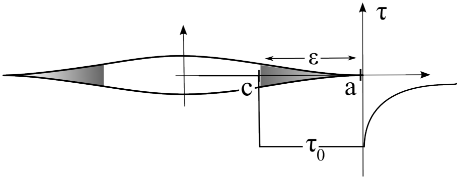
$$u(x, 0^+) = \frac{\tau_\infty}{\mu}\sqrt{x^2 - a^2}$$

The lower lengthscale problem

Cohesive zone

With crack face interaction:

- **cohesive zone** size $\epsilon = a - c$
- **cohesive stress** τ_0

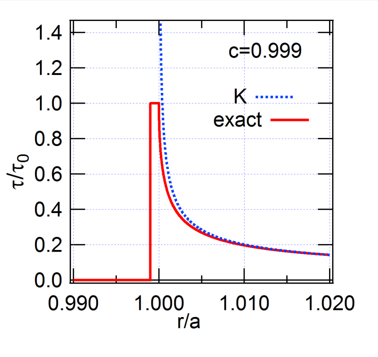


Cohesive stress and singularity regularization

Barenblat-Dugdale model (Maugis (2000))

Regularization of the stress singularity

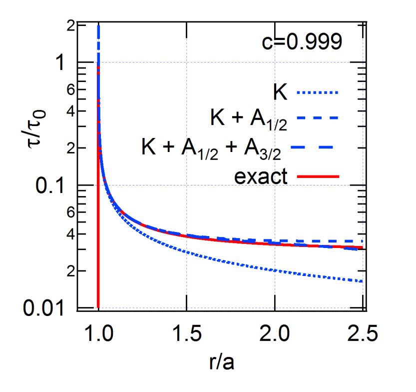
$$\tau(x,0^+) = -\frac{2}{\pi}\tau_0 \arctan\left(\frac{c\sqrt{x^2-a^2}}{x\sqrt{a^2-c^2}}\right)$$



Cohesive stress and singularity regularization

Barenblat-Dugdale model (Maugis (2000))

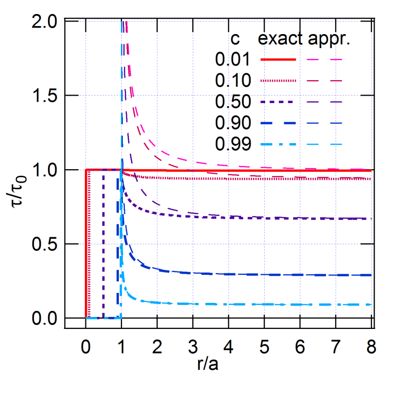
Regularization of the stress singularity



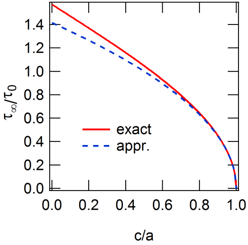
Far field and Williams expansion

$$\frac{\tau_\infty}{\tau_0} \simeq \sqrt{\frac{2\epsilon}{a}}$$

Downscaling



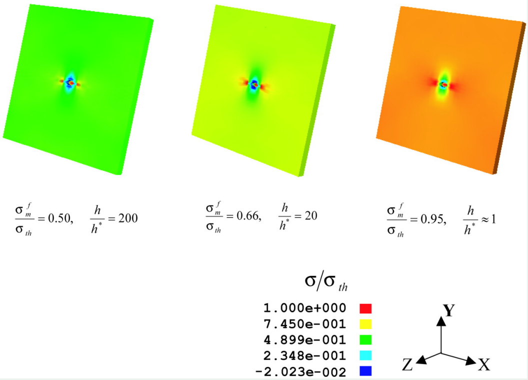
impact of cohesive zone size on far field



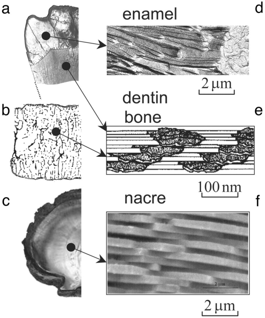
stress ratio

Far field stresses converge to cohesive stresses as size shrinks (**size effect**) – far field is not so far...!

Example – Bone toughness



Stress distribution

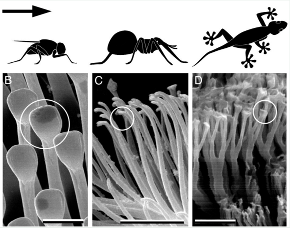


Biomaterial structures

Gao et al. (2003)

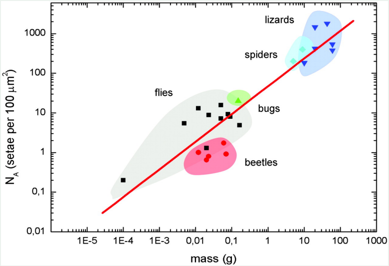
Adhesive contact

Animal pad division



Various pads as a function of species.

Size Effect

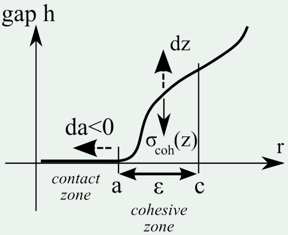


Pad division as a function of weight.

Arzt et al. (2003)

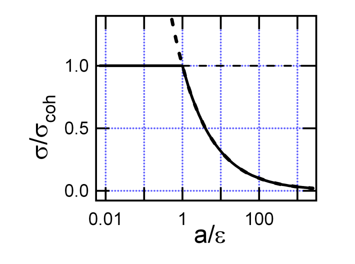
The lower lengthscale problem

Animal pad division



Cohesive zone.

Average stress



Cut-off with size reduction.

Arzt et al. (2003)

Energy balance

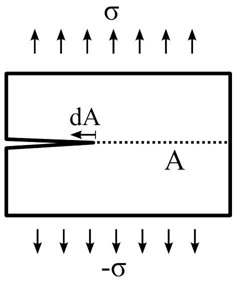
During crack propagation

energy release rate

$$\mathcal{G} \equiv \left. \frac{d\mathcal{E}_{el}}{dA} \right|_{\delta}$$

At equilibrium

$$\boxed{\mathcal{G} = \Gamma_0}$$



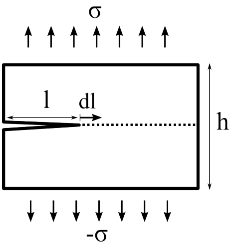
A crack with some remote loading.

Example – a crack in a thin plate

Energy balance

$$\mathcal{G} = h\frac{\sigma^2}{2E}$$

- elastic modulus E
- depth b



A crack traveling through a plate.

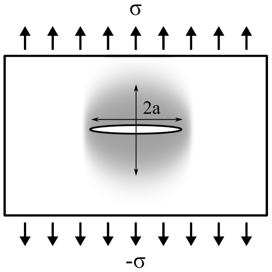
Energy release rate – the general case

Full 3D fracture

Energy release rate:

$$\mathcal{G} = \psi \frac{\sigma^2 a}{E}$$

where ψ is a numerical constant of the order of 1



A 3D crack – half-penny.

$$\boxed{\sigma \simeq \sqrt{\frac{Ew}{a}}}$$

Griffith (1921)

Crack tip energy flux

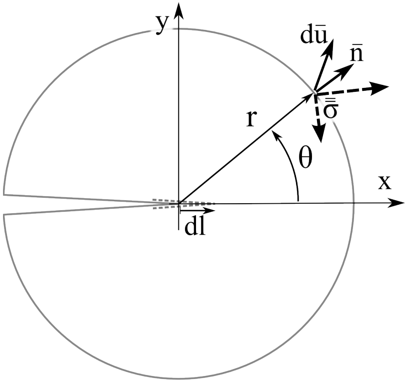
$$\mathcal{G} = -\int_{\mathcal{S}} \frac{d\bar{u}}{dl} \cdot \bar{\bar{\sigma}} \cdot \bar{n}$$

From pages 18 and 19 we have

- $\tau \propto r^{n-\frac{1}{2}}$, $n \geq 0$
- $u \propto r^{n+\frac{1}{2}}$

The **only non vanishing term** as $r \to 0$ is $n = 0$ (**the K term**) and

$$\boxed{\mathcal{G} = \frac{\pi K^2}{4\mu}}$$

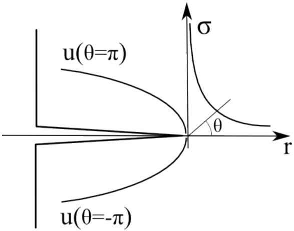


The energy carrying field is the singular term.

Energy release rate – crack closure method

$$
\begin{aligned}
dU_{el} &= \frac{b}{2}\int_0^{da} \sigma\,[u(\pi) - u(-\pi)]\,dr \\
&= \frac{bK^2}{2\mu}\int_0^{da}\sqrt{\frac{da - r}{r}}\,dr
\end{aligned}
$$

$$\mathcal{G} = \frac{\pi K^2}{4\mu}$$



Crack tip fields

with $r = da\sin^2\alpha$ and $\mathcal{G} = \frac{dU_{el}}{dA} = \frac{dU_{el}}{bda}$

Energy release rate – cohesive model

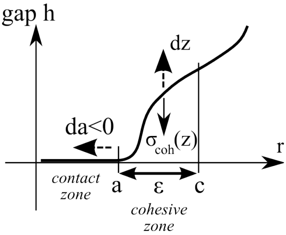
From the cohesive stresses

- steady state

$$\frac{d\mathcal{E}}{da} = -\int_a^\infty \sigma(z)\frac{dz}{dx}dx$$

- change of variable (cf p. 6)

$$\frac{d\mathcal{E}}{da} = -\int_0^\infty \sigma(z)dz = \Gamma_0$$

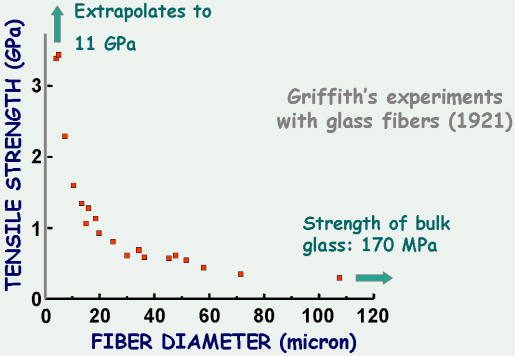


Schematics of the cohesive zone

Contribution from the cohesive stresses

Size effects in rupture

Glass

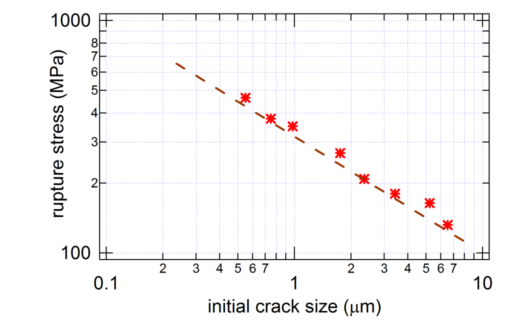


Tensile strength – Griffith's data.

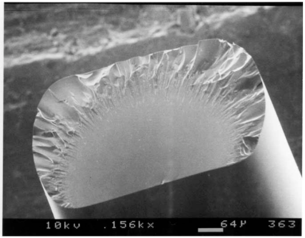
rupture is determined by the **surface flaw** size – Griffith (1921)

Impact of controlled flaw size

silica rupture

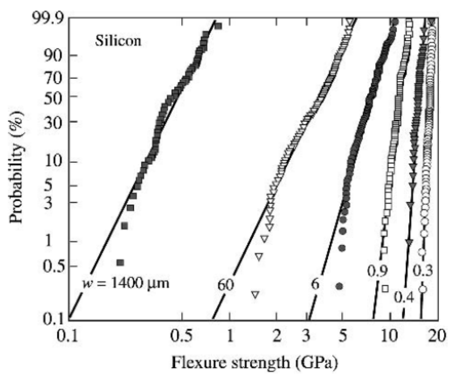


Strength as a function of flaw size - silica



Semjonov and Kurkjian (2001)

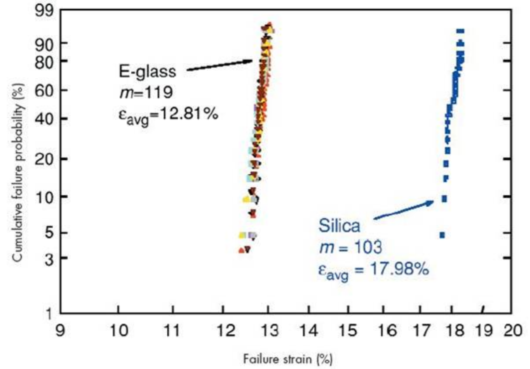
Semi-brittle materials



Failure distribution as a function of size of Si beams.

Namazu et al. (2000)

Ultimate tensile strain



Rupture strain distribution for glass and silica fibers.

Brow et al. (2005)

Strength vs. yield stress map

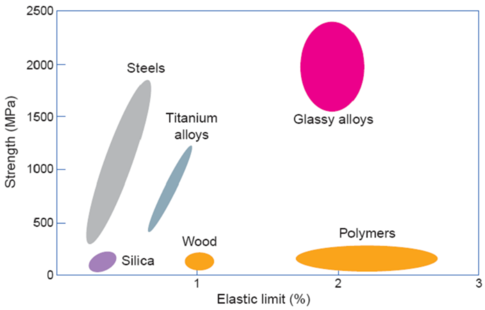


Fig. 3 Amorphous metallic alloys combine higher strength than crystalline metal alloys with the elasticity of polymers.

Strength distribution as a function of "elastic limit" for various materials.

Telford, Materials Today, March 2004.

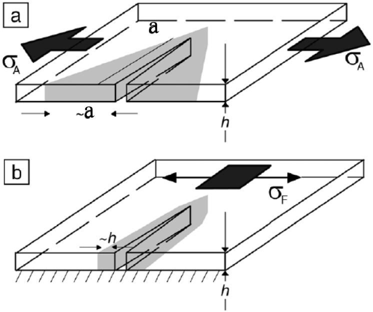
Substrate constraint on **thin film** cracking

Energy release rate

$$\text{a)} \quad \mathcal{G} = \psi_0 \frac{\sigma^2 a}{E}$$

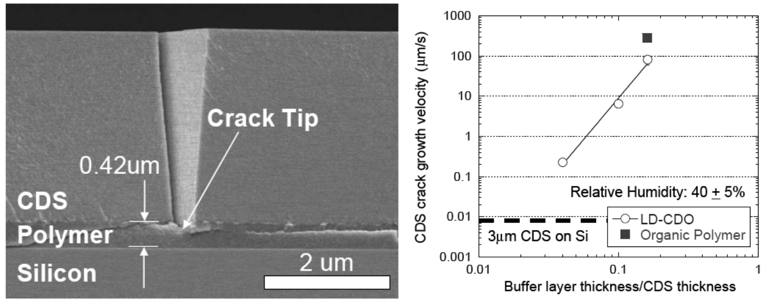
$$\text{b)} \quad \mathcal{G} = \psi_1 \frac{\sigma^2 h}{E}$$

Cook and Suo (2002)



Substrate constraint on thin films.

Impact of substrate constraint – compliant interlayer

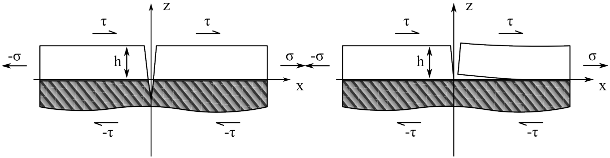


Cross section (left) and crack velocity (right)

Tsui et al. (2005)

Crack branching – the Cook Gordon mechanism

$$\sigma = \sqrt{\frac{E^{\star}\Gamma_{0coh}}{\pi h}} \quad \text{and} \quad \sigma = \sqrt{\frac{4E\Gamma_{0int}}{h}} \tag{1}$$



Branching criterion for coating fracture.

Interface delamination

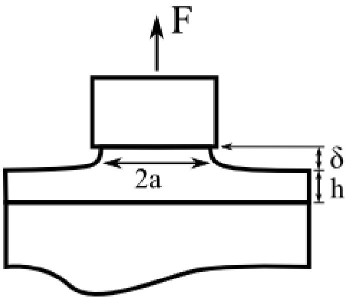
$$\Gamma_{0coh} > 4\pi\Gamma_{0int}$$

Pulling out a punch on a film

$$F = \pi a^2 E \left(\frac{d}{h}\right)$$

$$\mathcal{E} = \pi a^2 h \times \frac{1}{2} E \left(\frac{d}{h}\right)^2$$

$$\mathcal{G} = \frac{\partial \mathcal{E}}{\partial \pi a^2} = \Gamma_0$$



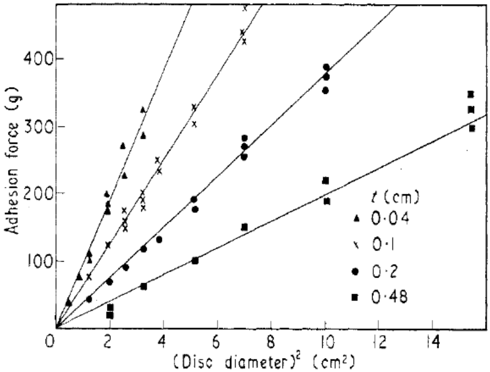
Rupture

- displacement:

$$\Gamma_0 = \frac{Ed^2}{2h}$$

- mean stress

$$\sigma = \sqrt{\frac{2E\Gamma_0}{h}} \quad (2)$$



The glue salesman paradox (Kendall (2001))

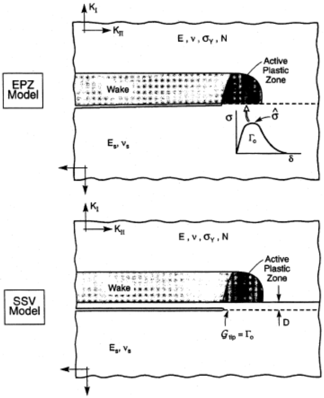
The less glue the more it sticks (*ie* the larger the pull-out force)

Rupture and macroscopic plasticity

- **plastic dissipation** contributes to the (steady state) **effective toughness** Γ_{ss}
- extends over radius R_{ss}
- yield stress:

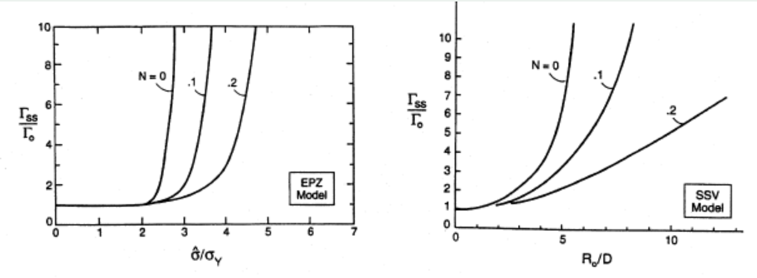
$$\sigma_y \simeq \sqrt{\frac{\Gamma_{ss} E}{R_{ss}}} \qquad (3)$$

Wei and Hutchinson (1999)



Two models for plastic dissipation

Plastic process zone

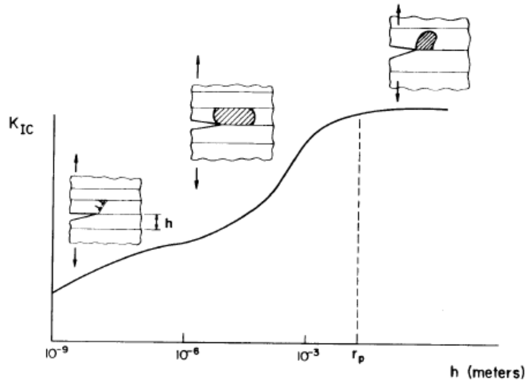


Toughness as a function of peak stress.

Wei and Hutchinson (1999)

Plastic dissipation in **thin film** delamination

Toughness as a function of **confinement**



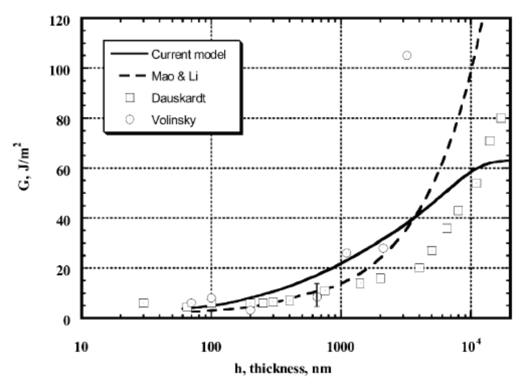
Three regimes of confinement.

Hsia et al. (1994)

- Cu film
- Mao model based on Hsia et al. (1994)
- Present model based on:

$$\sigma_y = \sigma_{y0}\left(1 + \frac{\beta}{\sqrt{h}}\right)$$

Contribution of plastic dissipation

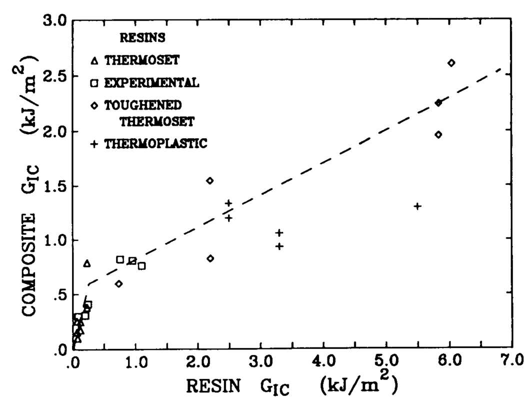


Interfacial toughness as a function of film thickness

Volinsky et al. (2002)

$$R_p = \left(\frac{K}{\sigma_y}\right)^2$$

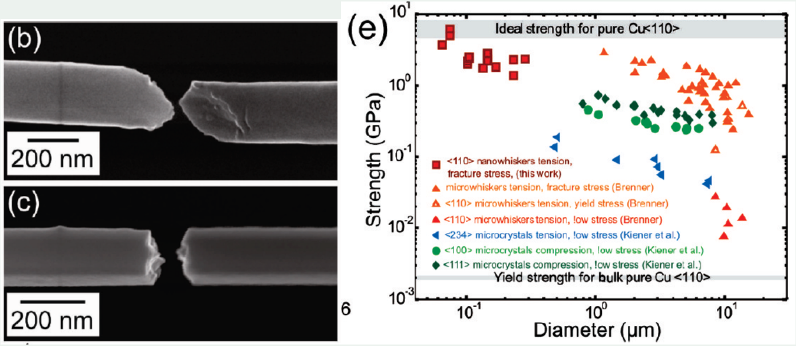
Composite toughness



Toughness of a **composite** as a function of matrix toughness

Bradley (1991)

Size and strength – one example



Richter et al. (2009)

Tensile strength of Cu whiskers

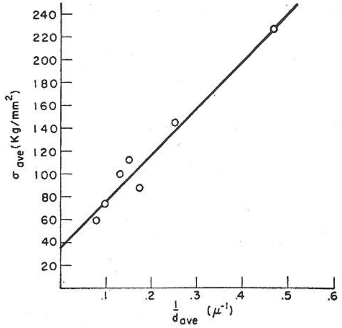


FIG. 10. The average strength of copper whiskers as a function of the reciprocal of the diameter.

Brenner (1956)

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In 1858, KARMARSCH* found that the tensile strength of metal wires could be represented within a few per cent. by an expression of the type

$$\mathbf{F} = \mathrm{A} + \frac{\mathrm{B}}{d} \quad \ldots \ldots \ldots \quad (22)$$

where d is the diameter and A and B are constants.

Griffith (1921)

Conclusion

Rupture

Beyond the physical rupture mechanisms at the interface

- intrinsically spans lengthscales
- intrinsically spans stress ranges
- involves specific material response

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